AN ANALYSIS OF A GEOMETRIC STABILIZATION SYSTEM USING BODY FIXED GYROS

STANLEY H. BUEG
AND
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Ser 2??
1 December [illegible]

[redacted]

From: Commanding Officer, U. S. Naval Reserve Officers Training Corps and Naval Administrative Unit, Massachusetts Institute of Technology, Cambridge, Massachusetts
To : Superintendent, U. S. Naval Postgraduate School

Subj: Theses; forwarding copies of

Ref : (a) Article 309, U. S. Naval Postgraduate School General Instructions, 1957

Encl: (1) Thesis, "An Analysis of a Geometric Stabilization System Using Body Fixed Gyros" by LT Stanley H. Bueg, USN and LT Francis E. O'Connor, Jr., USN (2 copies)
(2) CONF Thesis, "Performance of a Pendulous Integrating Gyro Accelerometer Under Environmental Conditions" by LT George W. East, USN (2 copies) C-43,541)

1. In accordance with reference (a), enclosures (1) and (2), theses prepared in partial fulfillment of the requirements for the Degree of Master of Science at the Massachusetts Institute of Technology for the members of the Aeronautical Engineering (Guided Missiles and Armament Control) (AR58) Group, are forwarded herewith. The delay in forwarding these theses to Postgraduate School has been occasioned by the security review backlog.

2. This letter is declassified when enclosure (2) is detached.

J. S. LEWIS

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By direction

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AN ANALYSIS OF A GEOMETRIC IDENTIFICATION SYSTEM USING BODY [illegible]

by

Stanley W. Bueg

Francis E. O'Connor Jr.

May 26, 1958

AN ANALYSIS OF A GEOMETRIC STABILIZATION SYSTEM USING BODY FIXED GYROS

by

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A.B., University of Missouri 1953

B.S., U.S. Naval Postgraduate School 1957

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B.S., United States Naval Academy 195[illegible]

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SUBMITTED IN PARTIAL FULFILLMENT OF THE

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MASTER OF SCIENCE

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1958

AN ANALYSIS OF A GEOMETRIC STABILIZATION SYSTEM
USING BODY FIXED GYROS

by

Francis E. O'Connor Jr.

Stanley H. Bueg

Submitted to the Department of Aeronautical Engineering on May 26, 1958, in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

Geometric stabilization is the process of maintaining a set of reference directions in the presence of disturbances. Geometric stabilization systems are used in the control of guided missiles. Two methods of geometric stabilization use single degree of freedom integrating gyros to establish inertial reference direction. One method mounts the gyros on a stable platform which isolates them from most of the missile motion. A second method mounts the gyros directly on the missile and in effect makes the missile the stable platform. In this configuration angular motion of the vehicle as a platform is more severe. This may lead to significant errors in maintaining reference directions, which in turn leads to error in missile orientation.

Errors in missile orientation develop because of the non-commutativity of three dimensional finite rotations and the interfering effects of missile angular motion on gyro performance. An error in orientation can be determined by comparing the angular postition of

the missile as seen in inertial axes with the angular position as seen in body fixed axes. The angular velocity of the missile as seen in body axes can be determined by solving the missile performance equations. This angular velocity can be expressed in inertial axes by means of a standard transformation. The angular velocity can then be integrated in both coordinate systems and the resulting angles compared. Integrating gyro gimbal angles define the missile orientation for guidance. Comparison of gyro gimbal angle and missile angular position as seen in body axes determines another error in orientation.

The performance equations of the system are non-linear and therefore solutions were computed on a digital computer for particular situations. The solutions showed that a systematic increase in error developed with time when the missile was subjected to periodic interfering torques. This effect existed regardless of the control system dynamics. When theamissile was subjected to random interfering torques the errors existed but due to the statistical nature of the results no further conclusions were made since only a single run was made. Further investigation of the system using both random and periodic interfering torques is desirable.

Thesis Supervisor: Wallace E. Vander Velde

Title: Assistant Professor of Aeronautical Engineering

May 26, 1958

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Hamilton:

In accordance with the regulations of the faculty, we hereby submit a thesis entitled An Analysis of a Geometric Stabilization System Using Body Fixed Gyros in partial fulfillment of the requirements for the degree of Master of Science in Aeronautical Engineering.

Stanley H. Bueg

Francis E. O'Connor, Jr.

ACKNOWLEDGMENT

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The graduate work for which this thesis is a partial requirement was performed while the authors were assigned by the United States Naval Postgraduate School for graduate training at the Massachusetts Institute of Technology. The thesis was supported by D. S. R. Project 52-142 sponsored by the Air Force Ballistic Missile Division of the Air Force Research and Development Command through U. S. A. F. Contract 04-(647)-103.

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TABLE OF SYMBOLS

Symbol	Definition
A	Angle - derivatives with respect to time are denoted by dots over the symbol for the first two derivatives and by numbers over the symbol for higher derivatives.
C_d	Damping coefficient
CT	Characteristic time
D	Deviation - followed by quantity referred to.
d	Diameter of missile at roll engine
DR	Damping ratio
e	Signal voltage
F	Thrust
I	Moment of inertia
IR	Inertia Ratio
K	Any arbitrary constant - distinguished by subscript
l	Distance from missile center of gravity to motor pivot point
M	Torque
S	Sensitivity
Sg	Signal
t	Time
V	Vector
W	Angular velocity
W_n	Undamped natural frequency

Subscript Symbols

Symbol	Definition
B	Body - refer to components in body axes
gim	Gyro gimbal
I	Inertial - refers to component in inertial axes
IA	Gyro unit input axis
I-B	Body with respect to inertial
igu	Integrating gyro unit
int	Interfering
OA	Gyro output axis
pcs	Pitch control system
rgu	Rate gyro unit
rtg	Roll torque generator
sg	Signal generator
SRA	Gyro unit spin reference axis
tot	Total
ycs	Yaw control system

All physical quantities are expressed in English units except angles which are in radians.

OBJECT

This investigation was conducted to determine the effects of angular base motion interference on a geometric stabilization system using body fixed gyros. The primary concern was the ability of the system to maintain a reference orientation in the presence of errors produced by the effects of interfering angular motion on gyro performance and the effects of three dimensional finite rotations of the base.

CHAPTER I

INTRODUCTION

Recent scientific and engineering advances have added satellites and ballistic missiles to the kinds of vehicles used by man. The successful use of these vehicles requires a means of controlling their motion during a part of their flight to keep them on a previously selected path. Ballistic missiles and similar vehicles may operate above the earth's atmosphere. The extreme altitudes require rocket propulsion. The operating region of these vehicles and the propulsion system used gives rise to new problems in implementing the control process.

Control implies a means of constraining and directing the vehicle to follow the desired path from the departure point. It is the function of the control system to direct the application of the necessary constraints. The basic principles are applicable for the entire class of ballistic missiles including satellites.

Rocket propelled vehicles expend their entire store of chemical fuel during the first part of the flight. This is followed by a relatively long period of unpowered flight during the remainder of the trajectory. Control is applied during the powered part of the flight when the fuel is rapidly being converted into mechanical energy. The objective is to achieve the altitude and velocity necessary to place the vehicle on the desired trajectory at the moment of thrust termination. The velocity of the vehicle must be determined with respect to its magnitude and its orientation. In this respect it is possible to associate a

vector with the instantaneous velocity of the vehicle. The task of the control system is to identify the vehicle velocity vector and maintain its magnitude and orientation in a prescribed manner. Most important, the control system must cause the vehicle to attain the predetermined value of the velocity vector at thrust cut-off. This means that the purpose of the control system is to control the magnitude and direction of the velocity of the vehicle at thrust cut-off.

In order to control the orientation of the vehicle's velocity vector, the control system must (1) establish and maintain a reference orientation in some usable form (2) provide a means for comparing the actual velocity vector orientation with the reference orientation and (3) provide a means for altering the velocity vector to make the deviation from the reference become zero. The reference orientation need not be constant during the flight. It may coincide with the desired flight path or with any other prescribed path. It is possible to distinguish between an inertial and an instantaneous reference orientation. The control system's function is to bring the velocity vector into coincidence with the instantaneous reference orientation.

It is desireable from the military standpoint to eliminate any dependence of the vehicle on external sources of information. There are systems that are not self-contained, requiring some of the components of the control system to be located on the ground or elsewhere outside of the vehicle. There are other systems that require external radiation or external reference points to establish the reference orientation. Inertial guidance systems have been constructed that are completely independent of external sources of information.[(1)*] These systems use

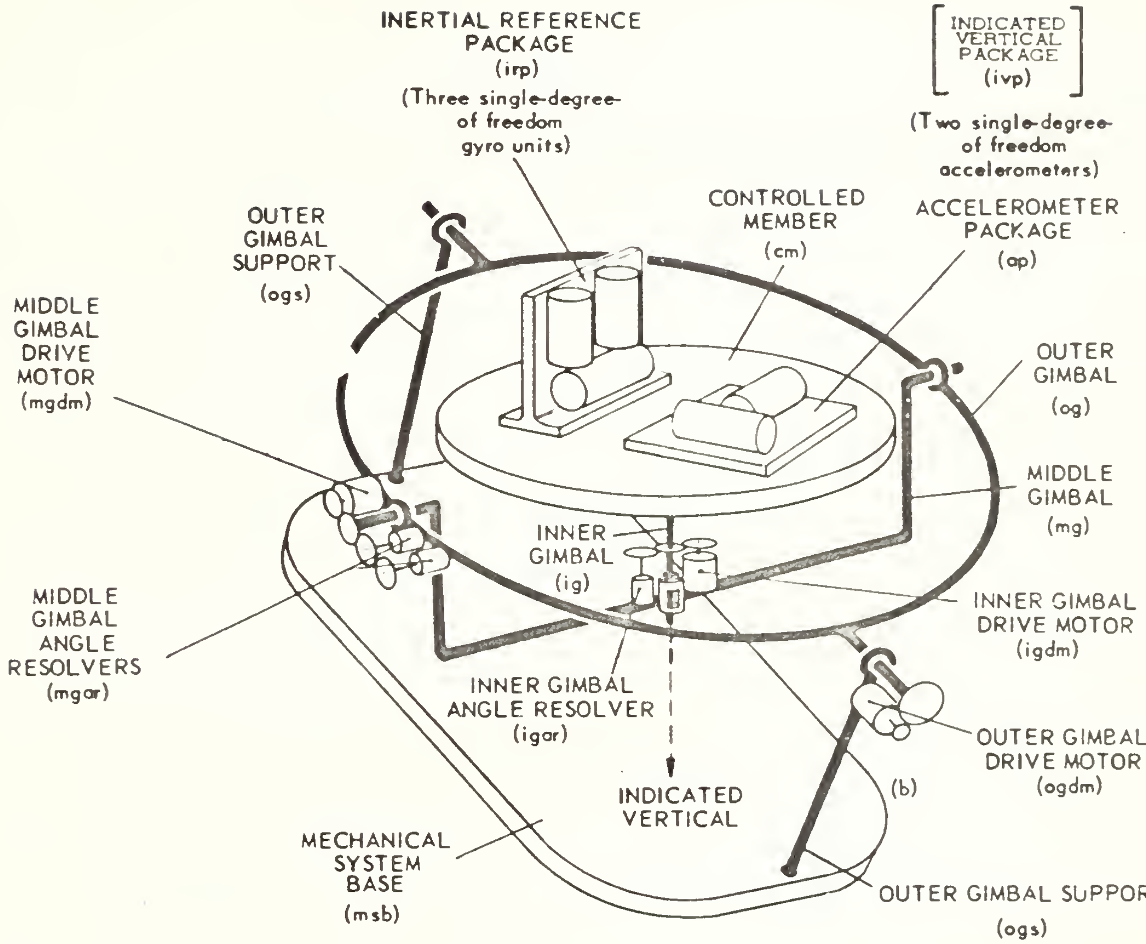
* Superscript numerals refer to similarly numbered references in the Bibliography at the end of this paper.

measurements made with respect to inertial space to establish the reference orientation and to measure deviations from the reference.

Since the control system depends on a process of relating the velocity vector of the vehicle to the reference orientation (now assumed to be in inertial space) any uncertainty in the representation of the reference, or in the information that corresponds to the representation, is equivalent to an uncertainty in the orientation of the velocity vector. The ability to control the orientation of the vehicle's velocity is impaired to the extent that the reference orientation is uncertain. To obtain high accuracy in controlling the vehicle trajectory, it is therefore necessary to maintain the reference orientation with high accuracy.

The most elementary task of the control system is to maintain the reference orientation when this orientation is fixed in space. Geometric stabilization [2] is the ability of the system to maintain the reference orientation in the presence of external sources of interference. The ability to track a changing reference orientation and to command the velocity vector to acquire a prescribed orientation with respect to the instantaneous reference is the guidance function of the control system. However a measure of the capability of the control system is its success in achieving geometrical stabilization.

Such a stabilization system has been instrumented using three single degree of freedom integrating gyros mounted on a gimbaled platform with their input axes along three mutually perpendicular directions. A line schematic diagram of a three axis gimbal system used for supporting such a platform is shown in Fig. 1-1.[3] The output signal of each gyro is proportional to the integral of the angular velocity of the platform about the respective axis. The outputs of these gyros actuate gimbal servo drive motors. These cause the platform to rotate so as to



NOTES: 1. THE BASE MOTION INTERFERENCE ISOLATION GIMBAL SYSTEM IS MADE UP OF THE OUTER GIMBAL SUPPORTS, THE OUTER GIMBAL, THE MIDDLE GIMBAL, THE INNER GIMBAL, THE ASSOCIATED DRIVE MOTORS AND THE ASSOCIATED RESOLVERS.

2. THE ELECTRICAL POWER SUPPLIES, ELECTRONIC UNITS, COMPUTERS, CONNECTIONS, RACKS AND OTHER COMPONENTS NECESSARY TO COMPLETE AN INERTIAL GUIDANCE SYSTEM ARE NOT REPRESENTED IN THIS FIGURE.

3. THIS ILLUSTRATION IS BASED ON FIG. 4 OF WRIGLEY, WOODBURY AND HOVORKA (2) AND FIG. 6 OF DRAPER AND WOODBURY (4).

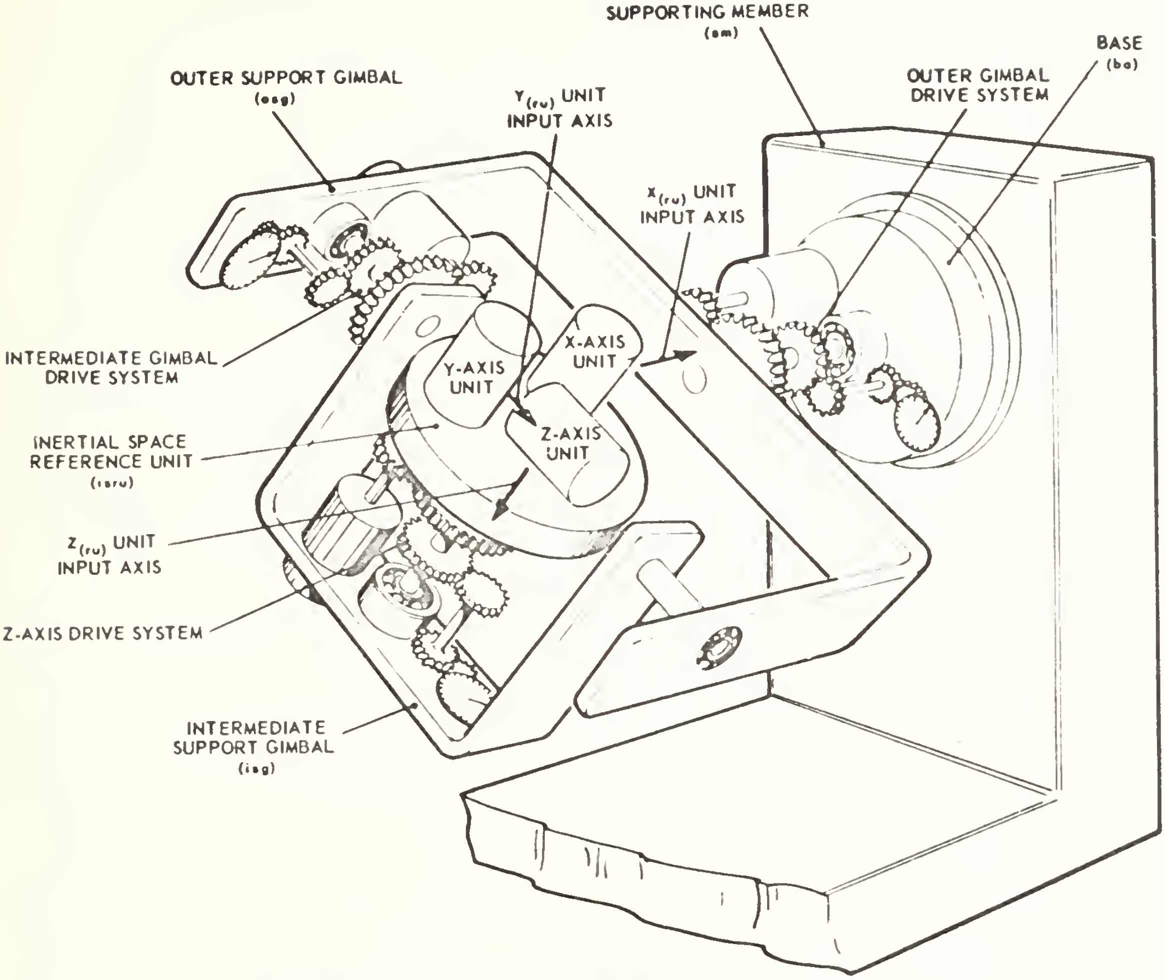
Fig. 1-1 .ine schematic diagram showing essential mechanical elements of inertial guidance system based on rotation of the inertial reference package with the indicated vertical.

null the gyro signals. At null indication of the gyro the platform is essentially restored to its original orientation in inertial space. Figure 1-2 is a pictorial diagram of one such instrumentation. Such a system may be considered as three, single axis space integrators (2) operating in parallel.

The essential features of a single axis space integrator are shown in the block diagram of Figure 1-3. This investigation was concerned only with the geometric stabilization problem. For the remainder of this paper it will be assumed that the command signal is zero, and the actuating input to the system is an interfering torque acting on the controlled member. The performance of a system possessing geometric stabilization may be described by the following steps: (1) The interfering torque acting on the controlled member causes an angular rotation with respect to inertial space about the gyro input axis. (2) The angular rotation causes a precession of the gyro with respect to the controlled member, producing a signal from the signal generator, which is proportional to the angle of precession. (3) The gyro signal is fed to the servo motor which them applies a torque to rotate the controlled member back to its initial orientation, nulling the gyro output signal. System damping is provided by compensation networks.

The basic single axis system then, consists of a gyro package mounted on a controlled member and a torque generator. The reference coordinate system is fixed in the controlled member. Therefore maintaining the controlled member orientation maintains the reference coordinate frame.

With the reference coordinate system established within the missile, the means of controlling the missile to its desired orientation within that frame may be considered. It is through control over the orientation of the missile, that control is exercised over the vehicle's



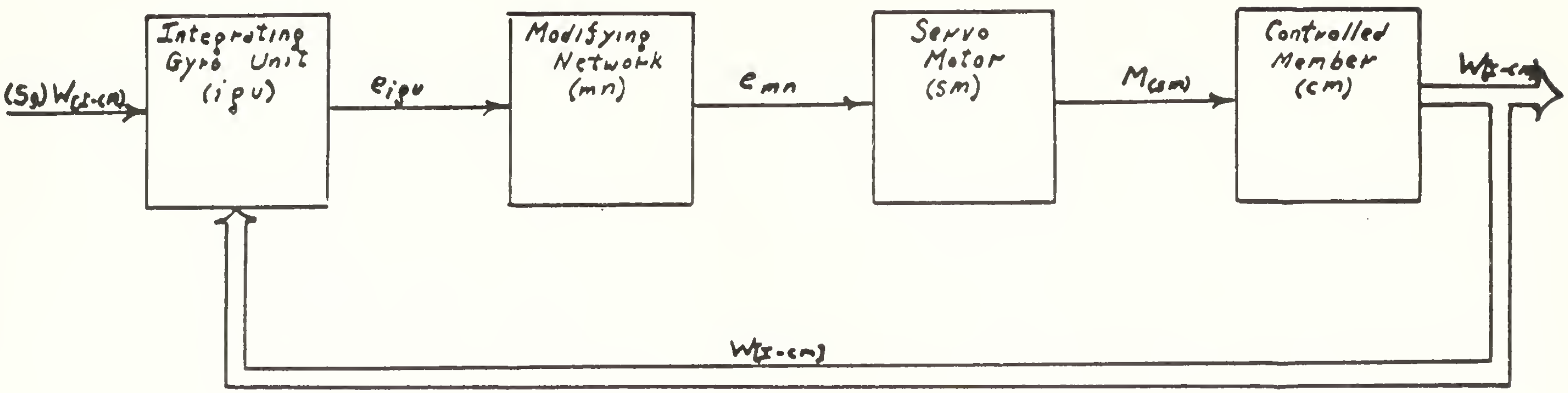
ELECTRICAL SLIP RINGS, CONNECTIONS, RESOLVERS ARE OMITTED FROM THIS FIGURE.

THE X-AXIS UNIT IS A SINGLE-AXIS INERTIAL SPACE ROTATION AND COMMAND SIGNAL RECEIVER WITH ITS INPUT AXIS ALONG $X_{(ru)}$.
THE Y-AXIS UNIT IS A SINGLE-AXIS INERTIAL SPACE ROTATION AND COMMAND SIGNAL RECEIVER WITH ITS INPUT AXIS ALONG $Y_{(ru)}$.
THE Z-AXIS UNIT IS A SINGLE-AXIS INERTIAL SPACE ROTATION AND COMMAND SIGNAL RECEIVER WITH ITS INPUT AXIS ALONG $Z_{(ru)}$.

This Diagram is based on Fig. 5 Of The Sherman M. Fairchild Publication Fund Paper No. FF-13, Institute of the Aeronautical Sciences, New York, January 1955. (Used With Permission)

Pictorial Diagram Of a Three-Axis Inertial Space Rotation Integrator Based On Three Single-Axis Inertial Space Rotation And Command Signal Receivers With Servo Drives For The Three Gimbal Axes.

Fig. 1-2



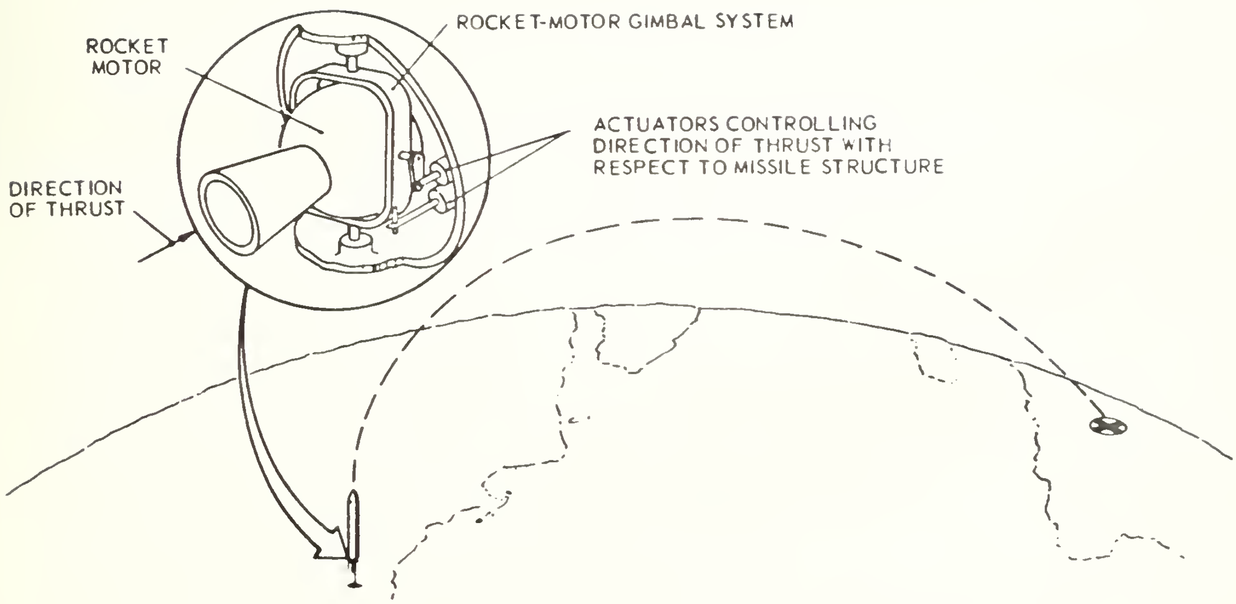
FUNCTIONAL DIAGRAM OF A SINGLE AXIS SPACE INTEGRATOR

Fig 1-3

velocity vector direction. In order to function, such a control system must include a means of generating the desired orientation of the vehicle in the reference frame, a means of measuring actual missile orientation, and a means of changing it to minimize or eliminate any deviations between the desired and actual orientation. Since this investigation is concerned with stabilization, it will be assumed that the missile has the ability of attaining the desired orientation and the function of interest in the control system, is that of maintaining that orientation in the presence of interfering torques applied to the missile.

One scheme for producing control torques for missile orientation, uses a thrust engine to produce restoring torques about the pitch and yaw axes of the missile and exhaust nozzles on the circumference of the airframe to control rotations about the roll axis. This requires that the thrust engine be mounted in gimbals, giving it two degrees of rotational freedom about the zero moment thrust line and a system for changing the engine orientation in response to command signals. Figure 1-is a diagram of such a configuration.

Looking at the combination of the missile control system and the inertial space reference system, there is a means of sensing deviations from desired orientation, a torque generator, and a controlled member in each system. In the inertial space reference system the gyro package senses deviations of the stable platform from its desired orientation. In the missile control system, the deviation sensed, is that of the actual missile orientation from that desired, both of which are specified relative to the stable platform. The stable platform is the controlled member of the inertial space reference system, and the missile is the controlled member of the missile control system. The inertial space



This Figure is based on Fig. 1-7 of "Inertial Guidance" (S) a monograph by C.S. Draper, W.W. Wrigley, and Sidney Lees, Published by the Instrumentation Laboratory, M.I.T., Cambridge, Massachusetts, August 1957. (Used with permission.)

A Diagram of a Missile With a Gimbaled Thrust Motor

Fig. 1-4

reference system uses servo motors as torque generators and the missile control system uses gimbaled thrust engines for that purpose. Elimination of the functional duplication of the combined systems can result in saving weight, and possible simplification of the over-all system.

Since inertial space reference is desired and it is the missile that is to be controlled, the units most reasonably eliminated are the stable platform with its servo drives, and the missile control system deviation sensing devices. This leaves the gyro package as the means of sensing deviations, the missile as the controlled member, and the missile control system as the torque generator. The principle of operation remains that of a space integrator, but the operating environment of the gyro package has been changed.

In the stable platform system for providing inertial space reference the gyros are subjected to only the motions of the platform. The assembly, including the gyros and gimbals, is subject to three inputs: (1) command signals (2) base motion (3) interference torques. Geometric stabilization exists when the gyro sub-assembly inside the gimbals maintains its orientation with respect to inertial space inspite of any base motion or the influence of directly acting interference torques. In a sense the assembly may be considered as a base motion isolation system. In particular it may be regarded as an active system to isolate the gyro sub-assembly against angular vibrations of the support. In a rocket propelled vehicle, where the vehicle generates intense vibration during the propulsion interval, the gimbal supported base motion isolation system greatly reduces the effect of the vehicular vibration. This enhances the ability of the inertial reference package to maintain the necessary reference orientation.

With the missile itself used as the controlled member, the gyros are rigidly attached to it. In this configuration the gyros are subjected to all the motions of the missile.

For operation outside the earth's atmosphere where aerodynamic forces, air turbulence and wind shear are absent, the missile structure is still subject to torques caused by lack of symmetry in the exhaust flow of the thrust engine, moments caused by the changing distribution of the propellant, and reactions on the frame when the thrust engine gimbal drive system moves the thrust line of the engine relative to the frame. Therefore, the angular vibration environment of the gyros which are rigidly attached to a missile frame is much more disturbing than when the gyros are attached to a stable platform within the missile.

The use of gyros rigidly mounted to the body of a missile subjects the gyros to all the motions of the missile. The effect of this disturbing environment, arising from angular motions of the missile, on the maintenance of an inertial reference frame is the subject of this investigation.

CHAPTER II

DISCUSSION OF THE PROBLEM

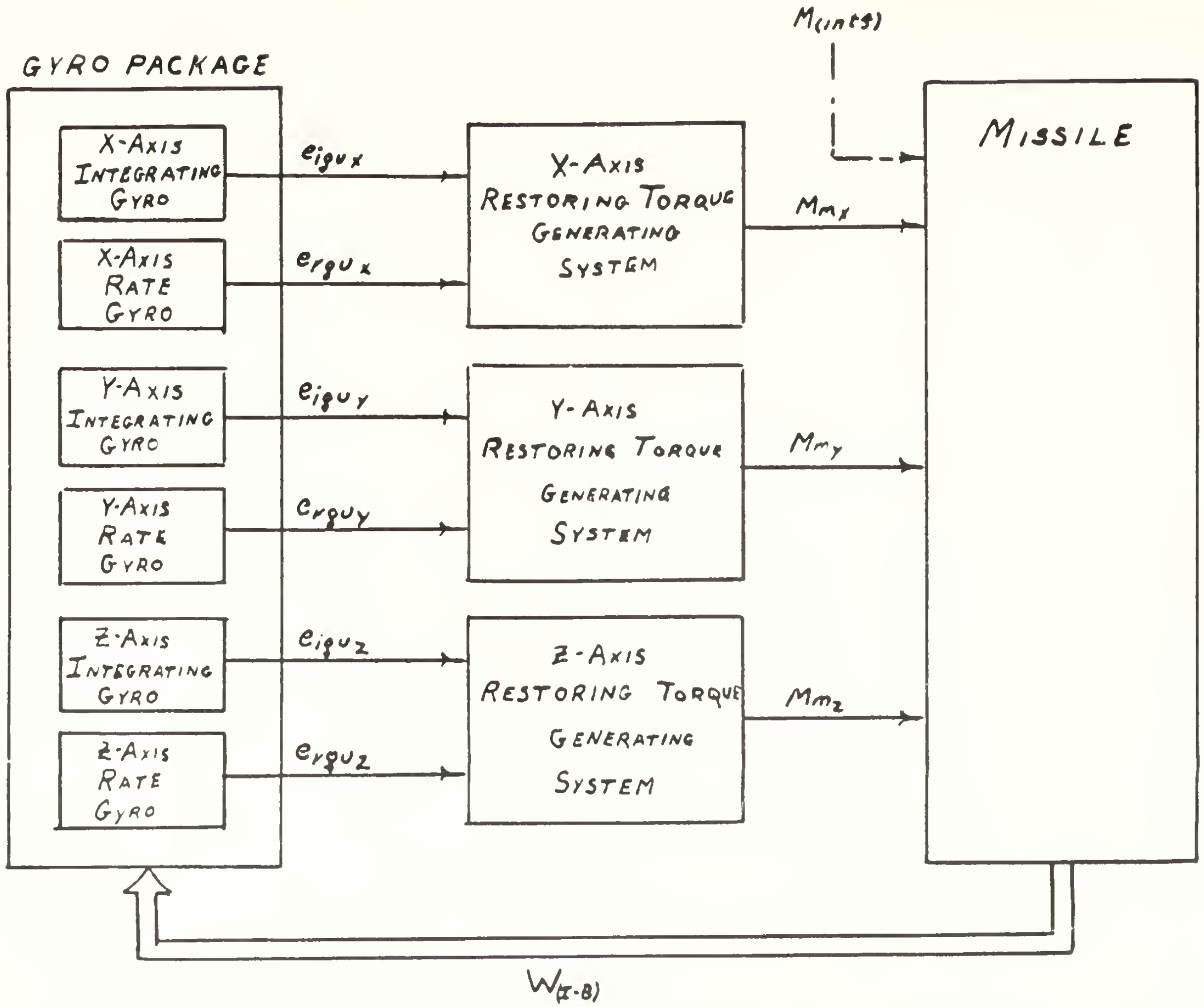
As was stated in the previous chapter, [illegible] stabilization systems which use body mounted gyros [illegible] not afford the isolation from angular vibration of the missile [illegible] systems employing a gimbal mounted [illegible]. The body fixed gyro system [illegible] interfering inputs [illegible] drive the controlled member (Fig. 2-1) from its commanded orientation. In the absence of any command signal to the system these interfering inputs may be considered as the only inputs to a system whose sole function is to maintain its initial orientation in inertial space. If in addition, the system is assumed to contain only ideal components so that component uncertainty plays no part in the analysis, it will be apparent that any error in orientation developed by the system must then be due to the effect of the interfering inputs. Such a model is ideally suited to [illegible] stated [illegible] in the [illegible] the paragraphs that follow.

System Operation

The [illegible] stabilization [illegible] elements. These are the controlled member, [illegible], and the torque generating system. The controlled member in the present case is the missile itself. This is assumed to be rigid body, so that the

orientation of the body is completely described by the orientation of its principal axes. The torque generating system provides the means for producing angular motion of the missile about its center of gravity in response to error signals developed by the gyro unit. The gyro package contains three single degree of freedom integrating gyro units rigidly mounted with their input axes perpendicular to the principal axes of the missile. The output signal from these gyros, is then proportional to the integral of the angular velocity of the missile about the input axes of each of the gyros. This provides a positional error signal for the torque generating system. In addition, the gyro package also contains three rate gyros mounted in a similar manner to provide a signal proportional to angular velocity for system damping. The system as described might be represented by the line schematic diagram of Fig. 2-1.

Referring to Fig. 2-1, the operation of the system can be described as follows: Base motion angular vibration can be represented as an interfering torque applied to the missile as shown. This torque will initially cause angular motion of the missile. The components of this angular motion along the gyro input axes will be sensed by the appropriate gyros which are rigidly attached to the missile airframe. The integrating gyros will then develop a signal proportional to the angular displacement of the missile about their respective input axes which will cause the torque generating systems to apply appropriate restoring torques to the missile. Similarly, the rate gyros generate signals proportional to missile angular velocity which are also applied to the torque generating systems to supply a damping torque to the system. In the simplest terms possible, where the gyro units and torque generators are taken as simple



GEOMETRICAL STABILIZATION SYSTEM USING BODY MOUNTED GYROS

Fig 2-1

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proportional mechanisms and the effects of geometric cross coupling between the missile equations of motion are neglected, such a system becomes a combination of three independent second order systems operated in parallel. Of course, this last statement is an oversimplification, and while it aids in visualization, the fact is, that system components do possess dynamic characteristics and there is also coupling between axes in any real system. Because of this, certain errors in system orientation are developed which are discussed in some detail later.

Three Dimensional Finite Rotations

Consider the response of the system described to a small disturbance. A set of inertial axes may be defined as coincident with the principal body axes of the missile before the onset of the disturbance. The motion of the missile relative to these inertial axes is to be determined.

The equations of motion for the missile can be conveniently written only in terms of body axes components, since it is only in body axes that the elements of the inertia dyadic for the missile remain constant. If such a set of equations are written, they make available upon their solution, the components in body axes of the angular velocity of the missile with respect to inertial space. The corresponding components in the inertial axis system will, in general, be different and can be determined by means of the standard matrix transformation for rotated axes systems. That is:

$$(2\text{-}1) \qquad \overline{W}_{(I-B)_I} = \overline{W}_{(I-B)_B} \cdot \overline{\overline{A}}_{(I-B)_I}$$

Where the subscripts I and B in the vector equation (2-1) refer to the components of the angular velocity vector as expressed in inertial axes and body axes respectively. The dyadic $\overline{\overline{A}}_{(I-B)_I}$ is an expression of the rotation matrix which is derived in Appendix A of this paper. It is made

up of elements involving the direction cosines relating the body and inertial axes set. In general these elements will depend on the order in which rotations about the three inertial axis occur. However, it is shown in Appendix A that for rotations which are small in the sense that the product of two angles is much less than one radian, he expression (2-1) becomes, in matrix form:

(2-2)

$$\begin{bmatrix} {}^{W}(I-B)_{X_I} \\ {}^{W}(I-B)_{Y_I} \\ {}^{W}(I-B)_{Z_I} \end{bmatrix} = \begin{bmatrix} 1 & -{}^{A}(I-B)_{Z_I} & {}^{A}(I-B)_{Y_I} \\ {}^{A}(I-B)_{Z_I} & 1 & -{}^{A}(I-B)_{X_I} \\ -{}^{A}(I-B)_{Y_I} & {}^{A}(I-B)_{X_I} & 1 \end{bmatrix} \begin{bmatrix} {}^{W}(I-B)_{X_B} \\ {}^{W}(I-B)_{Y_B} \\ {}^{W}(I-B)_{Z_B} \end{bmatrix}$$

where the subscripts X_I, X_B, Y_I, etc. identify the components of angular velocity (W) and the integrals of these components (A) in inertial and body axes respectively. The expression (2-2) is now independent of the order of rotation.

The expansion of (2-2) gives for the components of angular velocity along the X-inertial axis:

(2-3)

$${}^{W}(I-B)_{X_I} = {}^{W}(I-B)_{X_B} - {}^{A}(I-B)_{Z_I}\,{}^{W}(I-B)_{Y_B} + {}^{A}(I-B)_{Y_I}\,{}^{W}(I-B)_{Z_B}$$

Integrating this expression gives the angle through which the body has turned about the X-inertial axis:

(2-4)

$$\int {}^{W}(I-B)_{X_I}\,dt = \int {}^{W}(I-B)_{X_B}\,dt - \int {}^{A}(I-B)_{Z_I}\,{}^{W}(I-B)_{Y_B}\,dt + \int {}^{A}(I-B)_{Y_I}\,{}^{W}(I-B)_{Z_B}\,dt$$

The first term on the right in (2-4) is the angle through which the missile has turned about the X body axis. Obviously if the missile undergoes some arbitrary rotation and the system of Fig. 2-1 is successful in returning it to its initial orientation in body axes then the first term on the right in (2-4) goes to zero. Since the gyro elements which develop the error signal are mounted on body axes, this is precisely what the system attempts to do. This being the case, in order for the net rotation about the inertial axis to go to zero, it is required that the sum of the two remaining terms on the right in (2-4) go to zero. It would appear that this will not, in general, occur, and some error in orientation with respect to inertial space will develop in the system.

The effect demonstrated above, is essentially geometric in nature. It is an example of the noncommutativity of finite rotations and the analysis can be approached in this light. The effect has been observed in the performance of gyro test turntables and an analysis using this somewhat different approach has been made by Goodman and Robinson.(5)

The equation (2-4) may be interpreted from a slightly different viewpoint which has the virtue of indicating the nature of the error in orientation which may develop. Suppose that there are two observers; one in the inertial reference frame and one in the body reference frame. Each observer concentrates his attention on his own X-axis and is able to record rotation of the body with respect to inertial space about that axis. In addition the observer in body axes can cause rotation of the body with respect to inertial space about his X-axis. Now if the body is subjected to an arbitrary rotation the

observer in inertial axes sees a rotation about his X-axis which is the sum of the terms on the right in (2-4) while the observer in body axes sees a rotation which is equal to the first term on the right in (2-4). Let the body now be brought to rest at some time (t_1). Suppose now that the observer in body axes causes the body to rotate about his X-axis only, so as to make the net rotation as he has observed it zero. During this rotation both observers see exactly the same rotation. However, when the observer in body axes has finally driven the body so that the net rotation as he sees it is zero at some time (t_2), the observer in inertial axes still observes a finite net rotation. In fact, the observer in inertial axis sees a net rotation:

(2-5)

$$\int_0^{t_2} {}^{W}(I-B)_{X_I}\,dt = -\int_0^{t_1} {}^{A}(I-B)_{Z_I}\; {}^{W}(I-B)_{Y_B}\,dt + \int_0^{t_1} {}^{A}(I-B)_{Y_I}\; {}^{W}(I-B)_{Z_B}\,dt$$

$$= \int_0^{t_1} {}^{W}(I-B)_{X_I}\,dt - \int_0^{t_1} {}^{W}(I-B)_{X_B}\,dt$$

$$= \left[{}^{A}(I-B)_{X_I\,(t=t_1)} \quad - {}^{A}(I-B)_{X_B} \right]_{(t=t_1)}$$

$$= D\left[{}^{A}(I-B)_{X_I} - {}^{A}(I-B)_{Y_B} \right] \quad (t=t_1)$$

The advantage of the expression (2-5) becomes apparent when the concept of the control system of Fig. 2-1 brought into the discussion. The body axes set, the observer, and the means provided for rotating the body are exactly this control system. The time period ($0-t_1$) may be thought of as the dynamic time lag in the system and then the rotation observed by the observer in body axis is the

error signal in the system. This error signal itself represents an error in orientation of the body with respect to inertial space, but it is transient in nature. The system is aware of it and eventually, it will be driven out. However, there is in addition to this transient error, the deviation expressed in (2-5) which the system is not aware of and hence, can do nothing about. The advantage of the expression (2-5) is that it expresses the error in orientation due to the effects of finite rotations independently of any transient deviations in orientation due to system dynamics which may exist.

The determination of the magnitude of the error in orientation discussed above, and expressed in equation (2-5) is, at least in theory, straightforward. The equations (2-2) taken together with the performance equations of the system, form a set of equations which specify the motion of the missile, with respect to inertial space, in both the inertial and body coordinate frames. It is only necessary to solve these equations for the appropriate rotations and perform the subtraction indicated by (2-5), and the error in orientation is obtained. Unfortunately, these equations are non-linear, so that no solution in closed form for the general case is possible. However, for a specific system, solution by numerical methods is possible. Therefore, if a system which is more or less typical of those likely to be encountered in practice is specified, then useful information concerning the magnitude of error likely to arise in a practical system under the interfering effects of missile angular motion might be obtained. The specification of such a system and the analysis indicated are the purpose of the remainder of this paper.

Integrating Gyro Errors

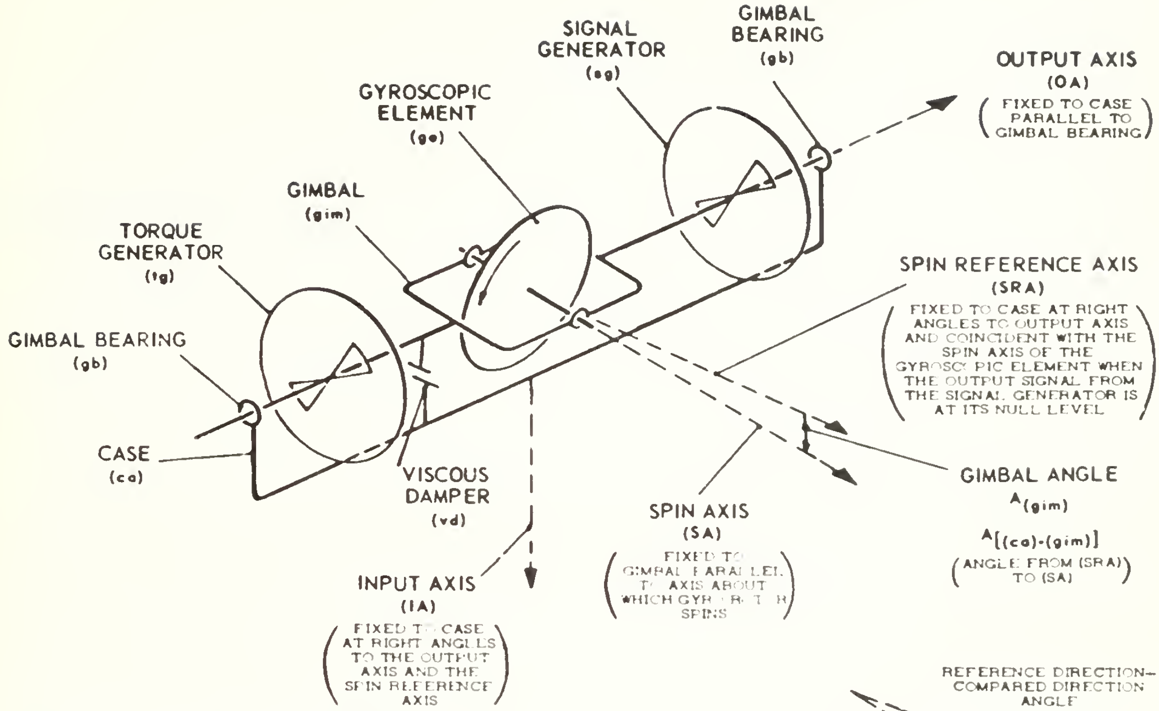
Before proceding with the specification of a system for analysis

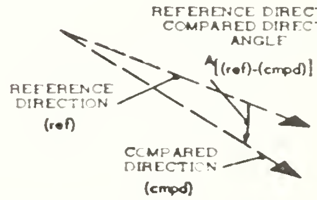
one other error in orientation which can develop in a system such as that shown in Fig. (2-1) should be discussed. This is essentially an instrument error occurring in the single degree of freedom integrating gyro. It is of interest because its effect on the system performance is, in some respects, remarkably like the effect due to finite rotations discussed above. This analysis will attempt to distinguish between the two effects and effect some comparison of their relative magnitudes.

For purposes of discussion, the geometric effect already described will be referred to as the finite rotation effect[5] while this new error producing effect will be referred to as kinematic drift.[7]

The origin of kinematic drift lies in the manner in which a single degree of freedom gyro performs its function. The principles of operation of these devices have been covered in detail elsewhere,[2,4,8] so that a simple functional description of their operation will suffice.

Fig. (2-2) shows a line schematic diagram of a single degree of freedom integrating gyro. As indicated in the figure, the main elements of the device are the spinning gyro wheel mounted in a gimbal, a viscous damper which acts between the gimbal and the case, and a signal generator which generates a signal proportional to the deflection of the gimbal with respect to the case. In operation an angular velocity of the case about the input axis (Fig. 2-2) causes a torque to be applied to the gimbal bearings which acts at right angles to the gyro spin vector. This torque will cause precession of the spinning wheel and its gimbal about the unit output axis. The viscous damper acting between the gimbal and case opposes this motion. The resulting angular velocity of the gimbal with respect to the case is proportional





NOTES:

1. POSITIVE SENSES SHOWN BY THE ARROWS ARE CHOSEN SO THAT (IA), (SRA), AND (OA) FORM A RIGHT-HANDED SYSTEM.
2. THE GYRO UNIT TEMPERATURE CONTROL POWER IS SUPPLIED TO A MOUNTING BLOCK ADAPTED TO RECEIVE THE GYRO UNIT CASE. THE FLOW OF POWER IS CONTROLLED BY THE DAMPER TEMPERATURE SETTING.
3. THE SYMBOL $A_{[(ref)-(cmprd)]}$ REPRESENTS THE ANGLE A MEASURED FROM THE REFERENCE DIRECTION ((ref) IN THE SUBSCRIPT) TO THE COMPARED DIRECTION ((cmpd) IN THE SUBSCRIPT).*

CASE – (ca) – THE STRUCTURE THAT GIVES SUPPORT FOR THE INTERNAL WORKING PARTS OF THE GYRO UNIT, ENCLOSES THE PARTS, AND CARRIES PROVISIONS FOR EXTERNAL CONNECTIONS OF ALL KINDS.

TORQUE GENERATOR – (tg) – COMPONENT FOR RECEIVING INPUT SIGNALS AND PRODUCING CORRESPONDING OUTPUT TORQUE APPLIED TO THE GIMBAL ABOUT THE OUTPUT AXIS.

DAMPER – (dmp) – SUBSYSTEM RECEIVING ANGULAR VELOCITY OF THE GIMBAL WITH RESPECT TO THE CASE AS ITS INPUT AND PRODUCING AS OUTPUT A RETARDING TORQUE ACTING ON THE GIMBAL ABOUT THE OUTPUT AXIS WITH A MAGNITUDE PROPORTIONAL TO THE MAGNITUDE OF THE ANGULAR VELOCITY OF THE GIMBAL WITH RESPECT TO THE CASE.

GYRO UNIT – (gu) – THIS ENTITY MADE UP OF THE COMPONENTS REPRESENTED IN THIS DIAGRAM AND ALL THE ADDITIONAL PARTS NECESSARY FOR A SINGLE PACKAGE TO CARRY OUT THE FUNCTIONS OF A GYRO UNIT.

SIGNAL GENERATOR – (sg) – COMPONENT FOR RECEIVING THE ANGLE OF THE SPIN AXIS WITH RESPECT TO THE CASE AS INPUT AND PRODUCING A CORRESPONDING SIGNAL THAT SERVES AS THE OUTPUT SIGNAL FROM THE GYRO UNIT.

GIMBAL – (gim) – STRUCTURE CARRYING THE BEARINGS FOR THE SPINNING ROTOR OF THE GYROSCOPIC ELEMENT, ROTORS FOR THE TORQUE GENERATOR AND SIGNAL GENERATOR, PART OF THE DAMPER, FLOAT SEALS AND STRUCTURE, BALANCE ADJUSTMENTS, STOPS, PIVOTS, ETC.

* A DISCUSSION OF GENERALIZED CONVENTIONS FOR SELF-DEFINING SYMBOLS OF WHICH $A_{[(ref)-(cmpd)]}$ IS AN EXAMPLE IS GIVEN BY DRAPER, McKAY AND LEES IN INSTRUMENT ENGINEERING [1], VOL. I.

THIS DIAGRAM IS BASED ON FIG. 13 OF THE SHERMAN M. FAIRCHILD PUBLICATION FUND PAPER NO. FF-13, INSTITUTE OF THE AERONAUTICAL SCIENCES, NEW YORK, JANUARY 1955. (USED WITH PERMISSION)

Line Schematic Diagram For The Single Axis Integrating Gyro Unit.

Fig. 2-2

to the angular velocity of the case with respect to inertial space about the input axis. Hence in a given time interval the gimbal will be displaced with respect to the case by an angle which is proportional to the displacement of the case with respect to inertial space in the same interval. The signal generator produces a signal proportional to gimbal displacement and therefore its output is proportional to the angular displacement of the gyro unit with respect to inertial space about the input axis.

The integrating gyro will perform as described above as long as the only rotation of the unit is about the input axis. However, when the gyro unit experiences rotations about the other two axes of Fig. (2-2) there are additional effects on the gimbal motion. These additional effects which are discussed in the following paragraphs, are the source of kinematic drift.

The precession of the gyro element under the influence of an input axis angular velocity carries the angular momentum vector of the spinning wheel with it. This produces a component of angular momentum perpendicular to the spin reference axis of the gyro unit. An angular velocity of the case about the spin reference axis will then also cause precession of the gyroscope wheel. The output signal of the gyro will therefore be in error by an amount proportional to the product of the sine of the gimbal angle and the angular velocity about the spin reference axis.

That is:

(2-6)

$$D(Sg)_{(igu)_{SRA}} = k_1 \int W_{(I-B)_{SRA}} \text{ Sin } A_{(gim)} \, dt$$

where in (2-6) the error is expressed as a deviation in output signal. The constant k_1 is a constant of proportionality and the angular velocity of the case with respect to inertial space is, as indicated by the subscript, about the spin reference axis of the gyro unit.

In addition to the signal deviation produced by the angular velocity about the spin reference axis a further deviation is introduced due to angular acceleration of the case with respect to inertial space about the output axis. Angular motion of the case is transmitted to the gimbal and gyro assembly through the fluid of the viscous damper. The viscous damper applies a torque to the gimbal which resists motion of the gimbal with respect to the case. If the gyro wheel is assumed non-spinning so that any torque due to it does not act, the torque summation on the gimbal becomes:

(2-7)

$$I_{gim} \quad \ddot{A}_{gim} \; - \; \ddot{A}_{(I-ca)OA} \quad = -c_d \quad \dot{A}_{gim}$$

where I_{gim} is the moment of inertia of the gimbal and gyro wheel about the output axis, $\ddot{A}_{(I-ca)OA}$ is the angular acceleration of the case with respect to inertial space about the output axis, and c_d is the damping coefficient of the fluid in the viscous damper. (2-7) simply expresses the motion of the gimbal assembly with respect to inertial space under the action of the single torque produced by the viscous damper. It may be rearranged to give the first order equation in gimbal angular velocity:

(2-8)

$$\frac{I_{gim}}{c_d} \ddot{A}_{gim} + \dot{A}_{gim} = -\frac{I_{gim}}{c_d} \ddot{A}_{(I-ca)OA}$$

From equation (2-8) it can be seen that the effect of a uniform angular acceleration of the case with respect to inertial space is to produce, in steady state, a constant angular velocity of the gimbal with respect to the case. (The transient period of this motion can be neglected in the present case because the characteristic time of the gyro, is extremely small compared to the other characteristic times of the system.) This slip between gimbal and case obviously appears in the output signal of the signal generator as a constantly increasing signal (for uniform acceleration). In terms of signal deviation where the ratio of gimbal inertia to viscous coefficient and the signal generator sensitivity are lumped to form a single constant:

(2-9)

$$D(Sg)_{(igu)_{OA}} = k_2 \int \ddot{A}_{(I-ca)OA}\, dt$$

The equations (2-6) and (2-9) may now be combined to give the total deviation of the gyro signal due to motion about the spin reference and the output axes. The resulting expression can be given the dimensions of angular displacement by dividing through by the signal generator sensitivity. This is desireable since it is necessary to relate this deviation to angular displacement of the missile. If this is done the total gimbal angle due to motion about all three axes becomes:

(2-10)

$$A_{gim} = S_{igu(W_B, A_{gim})} \int W_{(I-B)IA}\, dt \;+\; D(Sg)_{tot} \Big/ S_{(Sg)(A_{gim};e)}$$

where the second term on the right is the sum of (2-6) and (2-9) divided by the signal generator sensitivity. If the sensitivity of the integrating gyro unit from input axis angular velocity to gimbal

angular velocity is taken as unity then (2-10) may be written:

(2-11)

$$\frac{D(Sg)_{tot}}{S_{sg(A_{gim},e)}} = \dot{A}_{gim} - A_{(I-B)IA}$$

The similarity between this term and (2-5) will be noted. The gimbal angle in the case where input angular velocity and gimbal angular velocity are kept equal ($S_{igu(W_{I-BIA},\dot{A}_{gim})} = 1$) represents the angle through which the system is attempting to turn the missile about the gyro input axis. The second term on the right is the angle in body axes through which the body would be turned in the absence of gyro errors. Like the expression (2-5) the equation (2-11) eliminates the effects of any errors due to time lags in the system which will eventually be driven out. The effects of system dynamics are hence not included in this expression. For a given axis of the missile, say the X-axis, the input axis of the gyro controlling the axis and the body axis itself are parallel. Therefore the use of the axis system depicted in Fig. (2-2) will be discontinued and hereafter the gyro whose input axis is parallel to the X-axis of the missile will be referred to as the X-axis gyro. Also since the system really acts to control gyro gimbal angle and not body angle the signs in equation (2-11) will be reversed. This simply refers the error to gimbal angle instead of body angle. Equation (2-11) may then be written for the X-axis in form similar to (2-5) as follows:

(2-12)

$$\frac{D(Sg)tot}{S_{sg(A_{gim},e)}} = A_{(I-B)_X} - A_{(gim)_X}$$

$$= D(A_B - A_{gim})_X$$

The expressions (2-5) and (2-12) are now identical in form. They can be combined to give the total error in body orientation as it is represented by the gimbal angle. However, before expressing this deviation a word of caution is in order concerning the quantity A_{gim}. In succeeding discussion the reference angle for the system will be taken as the gyro gimbal angle, (A_{gim}). As was mentioned above if the gyro sensitivity from input axis angular velocity to gimbal angular velocity is unity, then this angle is the angle through which the system is currently attempting to turn the missile about the particular axis being controlled. In the present case this sensitivity is unity (10^4 integrating gyro).(9) Under these circumstances the total deviation of the body axes from the inertial reference fram after the effects of dynamic lags have been removed becomes the combination of (2-5) and (2-12):

(2-13)

$$D(A_I - A_{gim})_x = D(A_I - A_B)_x + D(A_B - A_{gim})_x$$

The expressions (2-13) along with the corresponding expressions for the other two axes of the system give the total current error in orientation in the system. In short they give expression to the difference in orientation between where the system should be going and where it is going.

In the preceding paragraphs two sources of error which can develop in a system such as the one shown in Fig. 2-1 have been discussed in some detail. The errors have been compared, and a total system error due to both effects has been expressed. It has been pointed out that the equations of motion describing such a system are non-linear and hence any attempt to determine the magnitude of these errors must

resort to numerical methods of solution. This means that a specific system must be chosen. The choice of such a system and the solution of the equations of motion for the system on the digital computer to determine the magnitude of the errors expressed in (2-5), (2-12), and (2-13) will be taken up in the remaining chapters of this paper.

CHAPTER III

THE SYSTEM

The analysis proposed in Chapter II requires, as a first step, the choice of a specific system such as the one shown in Fig. 2-1. In general this means that components, which are shown as blocks in Fig. 2-1, must be specified so that their performance equations may be written. In choosing these components it should be remembered that the errors under investigation will exist regardless of system dynamics. This was pointed out in the previous chapter. Therefore the specification of highly complicated system dynamics will add little to the investigation. On the other hand, it was desired to make the system at least representative of missiles which are stabilized in this manner. Hence the system described in the following paragraphs will attempt to include a minimum of dynamics in a system which might reasonably be expected to perform the required control function.

The system breaks down naturally to:

(1) The missile or controlled member.

(2) The torque generating system, by which is meant the mechanism that applies restoring torque to the missile in response to gyro signals.

(3) The gyro package, which includes both rate and integrating gyros.

These will each be taken up in turn. The necessary performance equations and the assumptions required to write them will be given. In addition, Appendix B of this paper presents the detailed derivation of these performance equations.

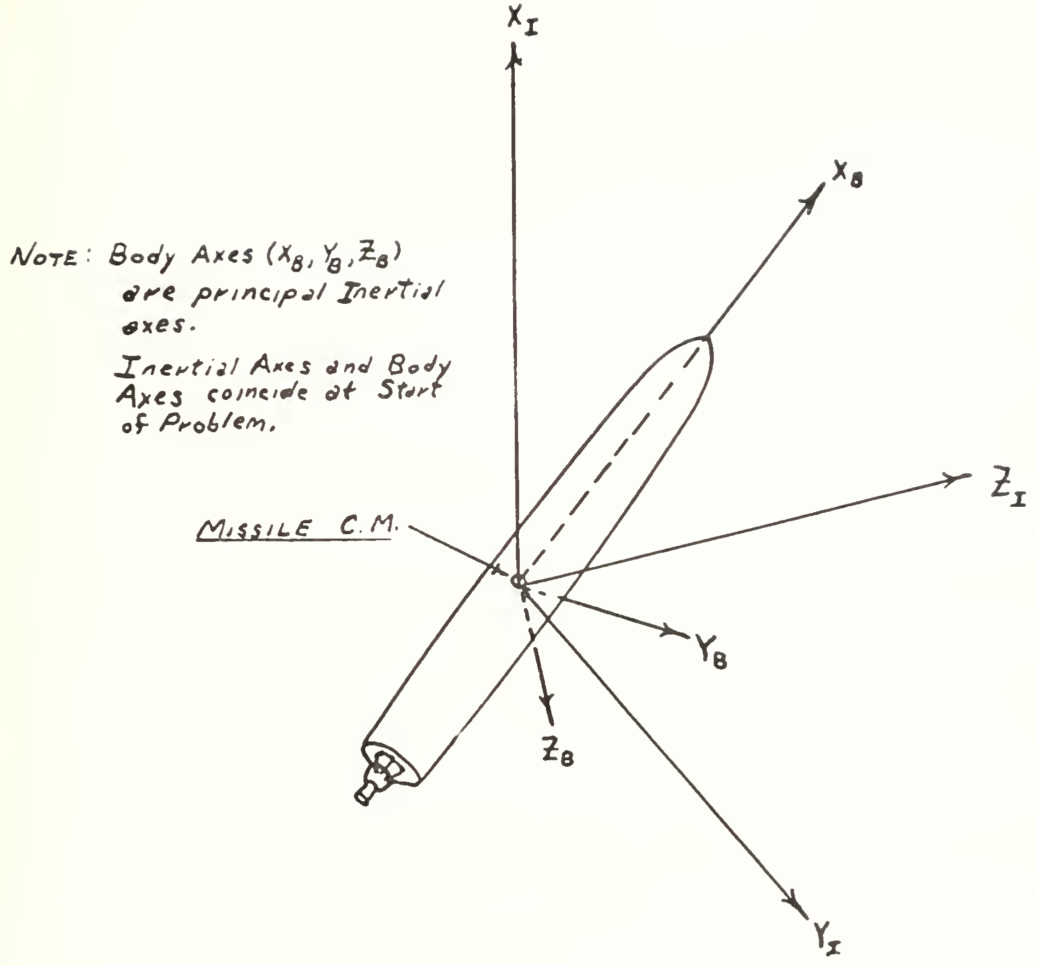
The Missile

The missile is assumed to be a rigid body. Its orientation in inertial space is completely described by the orientation of a set of orthogonal axes fixed in the body. These are taken as principal axes with origin at the missile center of gravity. Following the general aerodynamic convention[10] the X-axis is toward the missile nose, the Y-axis is perpendicular to the plane of the missile's trajectory, and the Z-axis lies in the plane of the trajectory. These axes will occasionally be referred to as roll, pitch, and yaw axes respectively. This axis set and the corresponding inertial set are shown in Fig. 3-1.

In an actual missile the burning of fuel causes a significant shift of the center of gravity and changes of the moments of inertia. These effects are not considered here. The center of gravity of the missile is assumed fixed and the missile moments of inertia are assumed constant.

Since the stabilization features of the control system are of interest, translation of the missile is not considered. The center of gravity of the missile is assumed fixed in inertial space.

With the above assumptions the equations of motion of the missile are easily obtained by summing moments about the missile center of gravity. In these equations the applied torques are the restoring torque (M_m) supplied by the system and the interfering torques (M_{int}) which represent base motion angular vibration. These equations[11]



MISSILE AXES SYSTEMS

Fig. 3-1

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(Euler's Equations) may then be written:

(3-1)

$$I_x\dot{W}_x - (I_y - I_z)W_yW_z = M_{m_x} + M_{int_x}$$

$$I_y\dot{W}_y - (I_z - I_x)W_zW_x = M_{m_y} + M_{int_y}$$

$$I_z\dot{W}_z - (I_x - I_y)W_xW_y = M_{m_z} + M_{int_z}$$

Where the moments of inertia I_x, I_y, I_z are principle moments of inertia of the missile. The angular velocities W_x, W_y, and W_z are the components in body axes of the angular velocity of the missile with respect to inertial space.

<u>The Torque Generating Systems</u>

The torque generating systems supply the required restoring torques about the three body axes of the missile. Fig. 3-4 shows a diagram of the mechanism by which this is accomplished.

The restoring torque in roll is supplied by auxiliary exhaust ports mounted on the circumference of the missile on opposite sides. The reaction of exhausting gas produces a couple about the roll axis. It is assumed that provision is included for reversing the thrust direction of these exhaust ports and for controlling the flow rate.

It is assumed in the present system that control of the roll torque generating system is proportional. That is, the system produces roll restoring moments in response to gyro input signals according to the following equation:

(3-2)

$$M_{m_x} = S_{rtg(e_{igu},M_m)}e_{igu} + S_{rtg(e_{rgu},M_m)}e_{rgu}$$

Where the sensitivities are the roll torque generating system's

sensitivities in producing roll moment in response to integrating gyro and rate gyro input voltages respectively.

The method for producing restoring torques in pitch and yaw is somewhat more complicated. The missile rocket motor is gimbaled to give it two degrees of rotational freedom about the Y and Z body axis. The rotation of the motor about these axes causes rotation of the thrust line so that it no longer passes through the missile center of gravity. This rotation of the thrust line produces the desired moment about the axis to be restored.

In the simplest form the mechanism for rotating the missile motor about the gimbal axes would be second order. It would provide sufficient torque in response to gyro signal to overcome the inertia reaction of the motor and gimbal. In addition, damping is required to prevent oscillation. Such a system would produce motor gimbal angle in response to gyro signal. If the engine thrust is assumed constant, the equation for restoring torques for small motor gimbal angles may be written. For pitch axis:

(3-3)

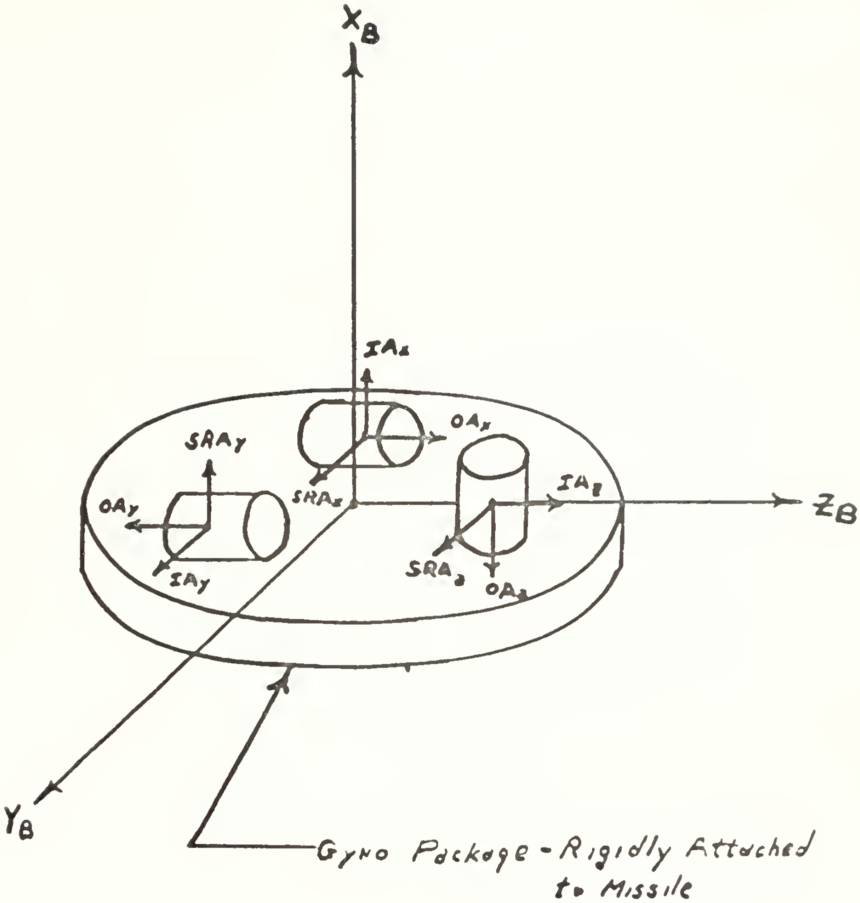
$$\frac{\ddot{M}_{m_y}}{W_{n_{pcs}}^2} + \frac{2(DR)_{pcs}\dot{M}_{m_y}}{W_{n_{pcs}}^2} + M_{m_y} = S_{pcs}(e_{igu}, A_{mg})F\,l\,e_{igu} + S_{pcs}(e_{rgu}, A_m)F\,l\,e_{rgu}$$

Where:

$W_{n_{pcs}}$ = Undamped natural frequency of the missile motor and its control system.

$(DR)_{pcs}$ = Damping ratio of the missile motor and its control system.

F = Engine thrust.



Rotation	Input Axis	Spin Ref. Axis	Output Axis
Roll	X_B	Y_B	Z_B
Pitch	Y_B	X_B	$-Z_B$
Yaw	Z_B	Y_B	$-X_B$

GYRO ORIENTATION

Fig 3-2

l = Distance from missile center of gravity to motor pivot point.

$S_{pcs(e_{igu}, A_{mg})}$ = Integrating gyro signal, to motor gimbal angle sensitivity of the pitch control system.

$S_{pcs(e_{rgu}, A_{mg})}$ = Rate gyro signal, to motor gimbal angle sensitivity of the pitch control system.

Appendix B presents the details of the mathematics leading to Equation 3-3 for both pitch and yaw control.

Gyro Package

The gyro package contains three single degree of freedom integrating gyros and three single degree of freedom rate gyros. Fig. 3-2 shows how these gyros are mounted with respect to body axes. Input, spin reference and output axes are shown for three gyros mounted on a base which is fixed to body axes. For the purposes of this discussion the orientations shown represent those of both the rate and integrating gyros.

The rate gyros are assumed to be proportional mechanisms. Angular velocity of the missile about the input axis produces a proportional voltage from the gyro at all times. The signal from these gyros is then of the form:

(3-4)

$$e_{rgu_x} = S_{rgu(W,e)} W_x$$

Where: $S_{rgu(W,e)}$ = The input axis velocity to output voltage sensitivity of the rate gyro unit.

The integrating gyros are assumed to be free of drift effects. Otherwise, the units are assumed to be identical to the unit described in Derivation Summary 2 of reference (2). Appendix B derives the

performance equation for the integrating gyro unit in terms of the gyro axes given in Fig. 2-3. These are then changed to give the equations in terms of body axes components for the orientation of the gyros shown in Fig. 3-3. The result for the X-body axis (roll) becomes:

(3-5)

$$(CT)_{igu}\ \ddot{e}_{igu} + \dot{e}_{igu} + W_y\ e_{igu} = S_{sg(A_{gim},e)}\ W_x + (CT)_{igu}\ \dot{W}_z$$

Where: $(CT)_{igu}$ = Characteristic time of the integrating gyro (App. B)

e_{igu} = Gyro output signal voltage.

$S_{sg(A_{gim},e)}$ = Gyro signal generator gimbal angle to voltage sensitivity.

Similar equations are given for the other two body axes in Appendix B.

The equations (3-1) through (3-5), along with similar expressions to cover all body axes, completely define the performance of the system. These might be combined in a straightforward manner to give a single performance equation for each axis of the missile. However in programming for the digital computer it is much simpler to treat the component performance equations as given above. Therefore no overall performance equation for the axes are written.

Appendix B gives a collection of the performance equations for all components for all three body axes. It is convenient to make certain changes of the variables in these equations at this point.

The equations of motion for the missile (3-1) about each axis are divided through by the moments of inertia of the missile about their respective axes. This changes the units of the equations from those of torque to those of angular acceleration. The ratios of restoring torque and interfering torque to the moment of inertia may then be

defined as new variables representing the angular acceleration of the body due to these torques acting alone.

The definition of angular acceleration due to restoring torque can now be carried into the equations for the torque generating system by dividing these equations through by the appropriate moments of inertia. This puts both the body and torque generating system equations in terms of angles and their derivatives.

The gyro equations can be expressed in terms of gyro gimbal angle by dividing through by the signal generator sensitivity. This change also carries through to the torque generating system. When all of the above changes have been made the overall sensitivity of the system appears as a coefficient in the equations of the torque generating system. Appendix B performs the changes of variable indicated and redefines the coefficients of the equations. The resulting set of equations along with the transformation equations (2-2) define a complete set for the problem under analysis. These are:

(3-6)

$$\dot{W}_x - (IR)_x W_y W_z = \ddot{A}_{m_x} + (S_{rcs(A_{gim},\ddot{A}_m)} A_{gim_x} + S_{rcs(W,\ddot{A}_m)} W_x$$

$$\dot{W}_y - (IR)_y W_x W_z = \ddot{A}_{m_y} + \ddot{A}_{int_y}$$

$$\dot{W}_z - (IR)_z W_x W_y = \ddot{A}_{m_z} + \ddot{A}_{int_z}$$

$$\overset{(4)}{A}_{m_y} + 2(DR)_{pcs} W_{n_{pcs}} \overset{(3)}{A}_{m_y} + W_{n_{pcs}}^2 \ddot{A}_{m_y} = W_{n_{pcs}}^2 S_{pcs(A_{gim},\ddot{A}_m)} A_{gim_y} + W_{n_{pcs}}^2 S_{pcs(W,\ddot{A}_m)} W_y$$

$$\overset{(4)}{A}_{m_z} + 2(DR)_{ycs} W_{n_{ycs}} \overset{(3)}{A}_{m_z} + W_{n_{ycs}}^2 \ddot{A}_{m_z} = W_{n_{ycs}}^2 S_{ycs(A_{gim},\ddot{A}_m)} A_{gim_z} + = W_{n_{ycs}}^2 S_{ycs(W,\ddot{A}_m)} W_z$$

$$(CT)_{igu}\ddot{A}_{gim_x} + \dot{A}_{gim_x} + W_y A_{gim_x} = W_x - (CT)_{igu}\dot{W}_z$$

$$(CT)_{igu}\ddot{A}_{gim_y} + \dot{A}_{gim_y} + W_x A_{gim_y} = W_y + (CT)_{igu}\dot{W}_z$$

$$(CT)_{igu}\ddot{A}_{gim_z} + \dot{A}_{gim_z} + W_y A_{gim_z} = W_z + (CT)_{igu}\dot{W}_x$$

$$W_{x_I} = W_x - A_{z_I} W_y + A_{y_I} W_z$$

$$W_{y_I} = W_y - A_{x_I} W_z + A_{z_I} W_x$$

$$W_{z_I} = W_z - A_{y_I} W_x + A_{x_I} W_y$$

Where: $S_{rcs(A_{gim},\ddot{A}_m)}$ = Gyro gimbal angle to restoring angular acceleration sensitivity of the roll control system.

$S_{rcs(W,\ddot{A}_m)}$ = Angular velocity to restoring angular acceleration sensitivity of the roll control system.

It will be noted that the equations (2-2) have been modified by dropping the subscripts indicating body axes components. This is in agreement with the system equations of motion. Angles and their derivatives which are referred to the inertial axes set are identified by the subscript (I). The sensitivities defined for the roll control system above have identical counter parts in the pitch and yaw control systems.

The constants required for the set of equations (3-6) must be specified. The complete set of these constants for the system are given in Table III-1. The inertia ratios were arbitrarily chosen. They represent what might be found in a missile of small to medium size such as the second stage of a two or three stage vehicle. The integrating gyro characteristic times are those for the MIT Instrumentation Laboratory 10^4 gyro. The natural frequency and damping ratio for the torque generating systems were arbitrarily chosen. The values

Constant	Value	Units
$(IR)_X$	0.1981	
$(IR)_Y$	0.8912	
$(IR)_Z$	-0.925	
$S_{(rcs)(A_{gim};\ddot{A})}$	-1.55	$/Sec^2$
$S_{(pcs)(A_{gim};\ddot{A})}$	-27.4	$/Sec^2$
$S_{(ycs)(A_{gim};\ddot{A})}$	-28.0	$/Sec^2$
$S_{(rcs)(\dot{W};\ddot{A})}$	- 3.11	/Sec
$S_{(pcs)(\dot{W};\ddot{A})}$	- 5.57	/Sec
$S_{(ycs)(\dot{W};\ddot{A})}$	- 5.44	/Sec
$W_{n_{pcs}}$	25	Rad/Sec
$W_{n_{ycs}}$	25	Rad/Sec
$(DR)_{pcs}$	0.5	
$(DR)_{ycs}$	0.5	
$(CT)_{igu}$	.003	Sec.

TABLE 3-1

TABLE OF SYSTEM CONSTANTS

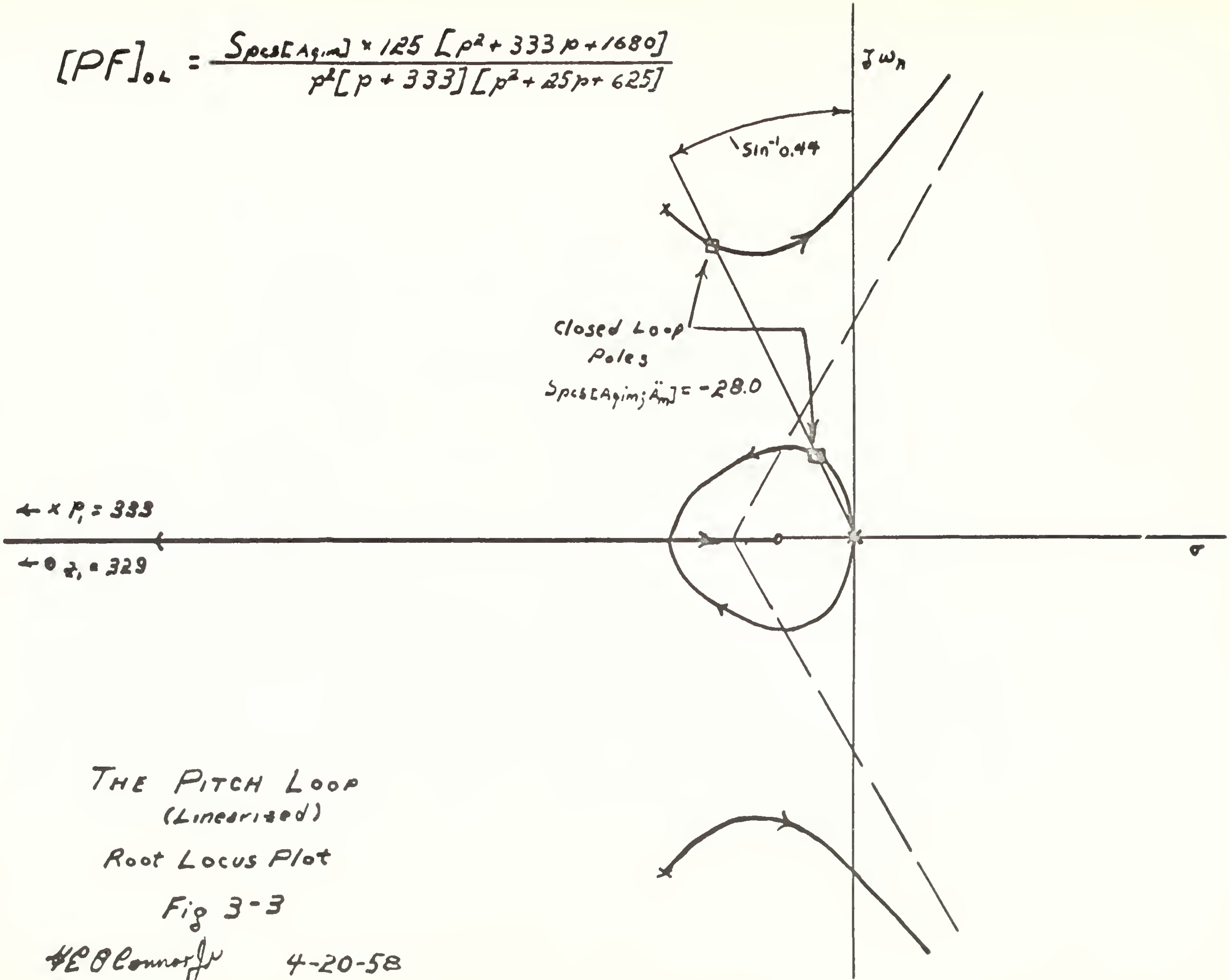
chosen are representative of what might be expected from a hydraulic servo in combination with the engine inertia. The system sensitivities which have all been combined in the torque generating systems remained to be chosen. These were chosen on the basis of a linearized stability analysis for the system which is discussed below.

In an effort to obtain some information on system stability, the equations of motion for one axis were combined. All coupling terms from the remaining axes were assumed to be zero. The root locus of the characteristic equation of the resulting fifth order system is shown in Fig. 3-3. The calculations leading to this locus are given in Appendix B.

The locus of Fig. 3-3 shows two oscillatory modes. As the open loop sensitivity is increased from zero, the pole pair associated with the torque generating system moves toward the right half-plane. At the same time the pole pair at the origin due to the integrating gyro and the missile performance equation move to the left. The system open loop sensitivity ($S_{pcs(A_{gim},\ddot{A}_m)}$) was chosen so as to give both pole pairs the same closed loop damping.

The preceding paragraphs have served to develop the performance equations of a simplified version of a geometrical stabilization system. It has been pointed out previously that the errors under investigation occur regardless of system dynamics. This fact can be demonstrated by varying the dynamics of the system just developed and solving the problem for the same interfering input. Since the problem was solved on the digital computer the variation in dynamics can be accomplished by rather elementary changes in the computer program. Therefore another system is proposed. The body equations of the set (3-6) were retained but the torque generating systems and integrating gyros were assumed

$$[PF]_{OL} = \frac{S_{pcs}[A_{gim}] \times 125\,[p^2 + 333p + 1680]}{p^2[p + 333][p^2 + 25p + 625]}$$



THE PITCH LOOP
(Linearized)
Root Locus Plot
Fig 3-3

H E O'Connor Jr 4-20-58

to have no dynamics. A set of equations similar to the set (3-6) is given for this system in Appendix B. The programming of these two systems for the digital computer is taken up in the next chapter.

CHAPTER IV

METHOD OF SOLUTION

Non linear differential equations are not in general amenable to solution in closed form.[12] Finding the exact solution of a number of simultaneous, non linear differential equations is even less probable. However, numerical techniques have been developed for obtaining particular solutions of differential equations, either linear or non linear. The solution so obtained will, as the name suggests, be in number form. For solution, the equations must be stated with numerical coefficients. The coefficients may be variable, but the manner in which they vary must be known. Certain initial conditions of the variables are also needed. The computation time required to solve for the dependent variables over a given range of the independent variable, will depend upon the numerical technique and the accuracy required. For improved accuracy the increment of the independent variable between calculated points must, in general be decreased, thus increasing computation time.

The numerical method used in solving the set of non linear differential equations (3-6) which are listed in the previous chapter is the Runge-Kutta Method with the Gill modification.[13] Use of this method requires a known value of each variable and the first derivative of each, at the start of each integration step. If the differential equation is higher than first order the integration process must be

used as many times as the order of the equstion. For example if $\frac{d^2y}{dt^2} = F(x, y, z, t, \frac{dx}{dt}, \frac{dy}{dt})$ the value of the second derivative of y is calculated at a point, say t_o, where values of x, y, their first derivatives, z, and t are known. The integration method will give the value of the first derivative of y for $t = t_o - \Delta t$ where Δt is the integration interval. To get the value of y at the point $t_o + \Delta t$ the first derivative, whose value was calculated for $t = t_o$ from the previous time step which started at $t - \Delta t$, is integrated using the same process. The equations which indicate the need for solving for lower order derivatives are called auxiliary equations. The eight second order differential equations mathematically describing the system, therefore require that the integration method be applied sixteen times at each time step. The transfer to inertial axes of rotation rates expressed in body axes give three more derivatives that must ve calculated and integrated for each time step. The equations, in computation form, along with their auxiliary equations, are shown in Appendix B.

The basic tool used in solving the differential equations, given in Appendix B was the I. B. M. 650 magnetic drum data-processing machine.(14) The unit used is operated by the Mathematics Group in the M.I.T. Instrumentation Laboratory. The machine used has an internal storage of two thousand registers, two tape units, floating point arithmetic, and index accumulators.

The problem was programmed using a floating address assembly routine, called Flad,(15) developed by the M.I.T. Instrumentation Lab Math Group. The basic advantages in using this assembly routine are the simplifications in coding and the optmization of the program to

reduce computation time. The differential equation routine was available as a subroutine requiring only entry and exit programming.

The program could be broken down into functional parts as follows:

1. Obtain the forcing function, $A_{(int)}$, for each axis.

2. Compute the values of the derivatives and store them in the proper registers to enter the differential equation subroutine.

3. Entry and exit instructions needed with the subroutine.

4. Put the required numbers in the desired form and read them out of the computer.

5. Start the cycle over again at the next time step.

In running the problem the time step needed to give the required accuracy of solution proved to be unfortunately small. The root locus of the linearized system, given in the preceding chapter indicated that the postulated system should be stable. In wringing out the program a sinusodal forcing function of 15 radians per second was used. In a linear system time increments of one twentieth of a period usually give accurate results. For such a time step the solution of the system equations diverged. Using one millisecond time steps showed that the solution was stable and some of the variables changed rapidly requiring that the time step be small for the digital solution to follow the changes. By trial and error it was found that for a time step of five milliseconds the computed values of the variables agreed with the same computed values when one millisecond was used. At ten millisecond time steps the solution diverged. Therefore five millisecond time steps were chosen. This time step resulted in approximatly twenty one hours of computing time being required to simulate one minute of problem time, using square pulse forcing functions.

Suggested methods of forcing the equations were by the use of actual acceleration data, summing sine waves, and using pulses. Use of data would require reading data cards at each computation point. This would have required the punching of at least four hundred cards for each second of problem time. Evaluating a sine takes 150 milliseconds. For a single sine wave per axis almost a half a second would therefore be required each time the forcing function was calculated. For a square pulse only additions and subtractions are required and little time is spent in the forcing function part of the program. The selection of square pulses used in this investigation was primarily determined by the computation time required.

The system was programmed to determine the response to three forcing functions.

1. An initial displacement of the missile about all three axes.

2. Periodic rectangular pulses of angular acceleration applied to all three axes.

3. Random rectangular pulses of angular acceleration acting on all three axes.

The initial displacement of the missile about all three axes was used to determine the transient behavior of the system. A similar initial angular displacement condition could be realized in an actual missile. For a multistage missile the separation process would be expected to cause some rotation of the next stage. When the propulsion system fired the control system would see the displacement as an initial error in orientation.

Rectangular pulses were used for periodic forcing of the system

primarily due to the speed with which they could be generated, as noted earlier. Use of these pulses added no appreciable computation time over that required for the initial displacement case. These pulses had a magnitude of four tenths radian per second squared. For each axis the pulse duration was a tenth of a second with two tenths of a second between the end of one pulse and the start of another. The pulses were phased so only one axis at a time was being forced. When the pulse was taken off of one axis and applied to another, its sign was changed. This gave the period of the forcing function's fundamental frequency as six tenths of a second.

The random pulses of angular acceleration were limited in magnitude from zero to ninety milliradians per second squared, in increments of ten. Pulse duration was limited to five hundred milliseconds in increments of five milliseconds. The sign, magnitude and duration of each pulse was determined using digits from registers containing calculated values of some variable. These registers contained values in floating point form. No digits less than two digits to the right of the first significant figure were used. These digits varied at every time step. The digits used to determine the forcing function for one axis came from variables primarily influenced by motions about the other axes. A single digit determined the pulse magnitude. To determine the sign of the pulse, five was subtracted from a single digit and the sign of the difference was used. The length of the pulse in five millisecond increments was determined using two digits. These procedures give a forcing function on each axis which is independent of the mode of forcing on the other two axes. The coupling between axes given by Euler's equations and the gyro equations

does not however make the angular motion about any one axis independent of the motions about the other two axes.

Programming for the further simplified systems which were mentioned in Chapter III was accomplished by removing parts of the original program.

CHAPTER V

DISCUSSION OF RESULTS

This chapter discusses the results obtained from the solution of the system equations on the digital computer. The discussion is concerned primarily with the angular differences developed in Chapter II. These differences represent errors in indication of body orientation. The difference between inertial angle and body angle is the error in orientation due to finite rotations. The difference between body angle and gyro gimbal angle is the error in orientation caused by gyro characteristics. The sum of these differences (A_I-A_{gim}) is the net error in orientation due to these effects. These differences do not include the deviation in orientation resulting from dynamic lags in the system.

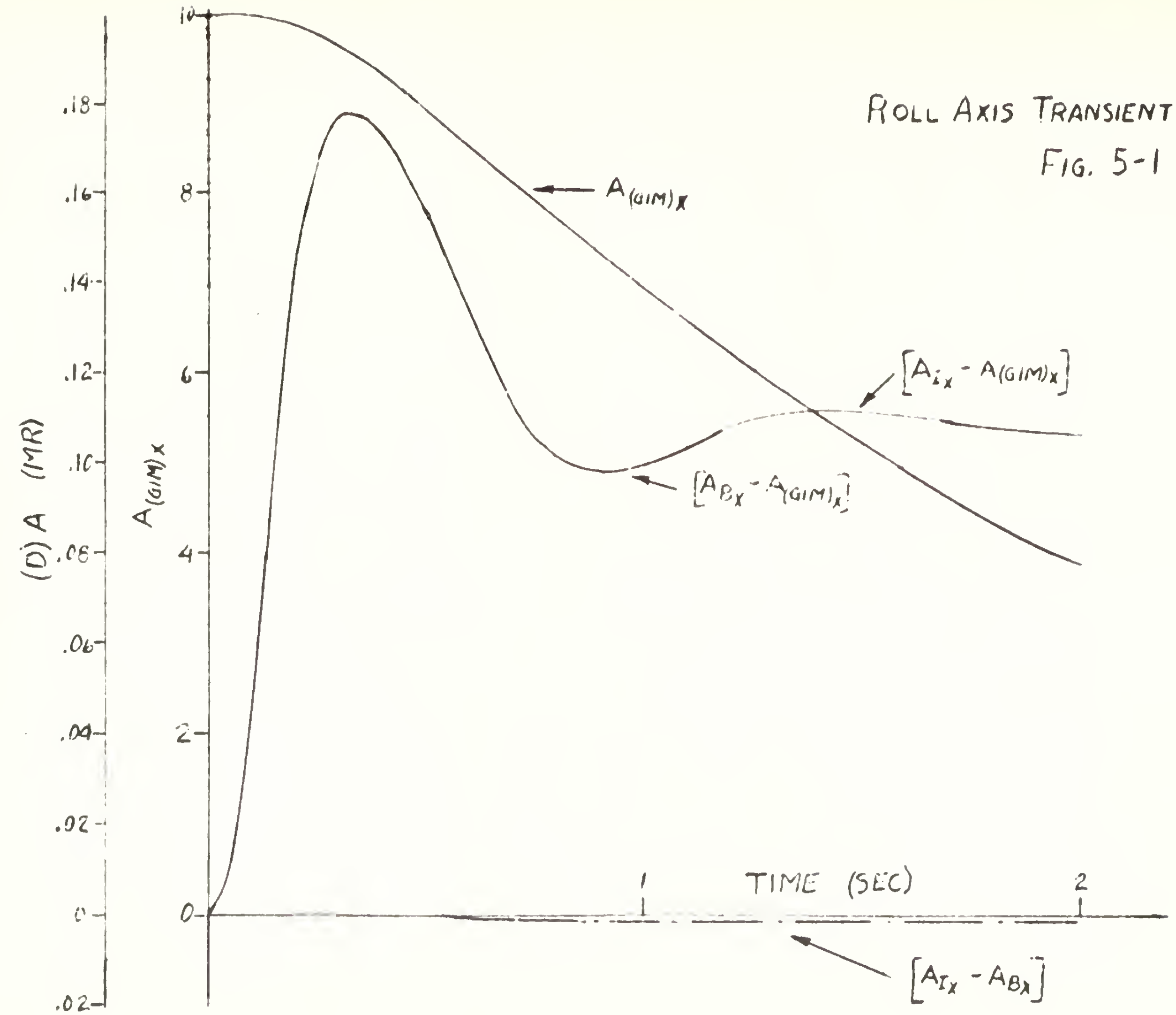
Transient Response

It was desireable that some examination of system stability be made. In order to do this the response of the system to an initial angular displacement was computed. No interfering angular accelerations were applied during this response.

Figs. 5-1, 5-2, and 5-3 show the response of the system in restoring from an initial angular displacement. In these figures the gimbal angle is taken as the reference variable. This was done because the gimbal angle is the variable which the system attempts to null. The figures

Roll Axis Transient Response

Fig. 5-1



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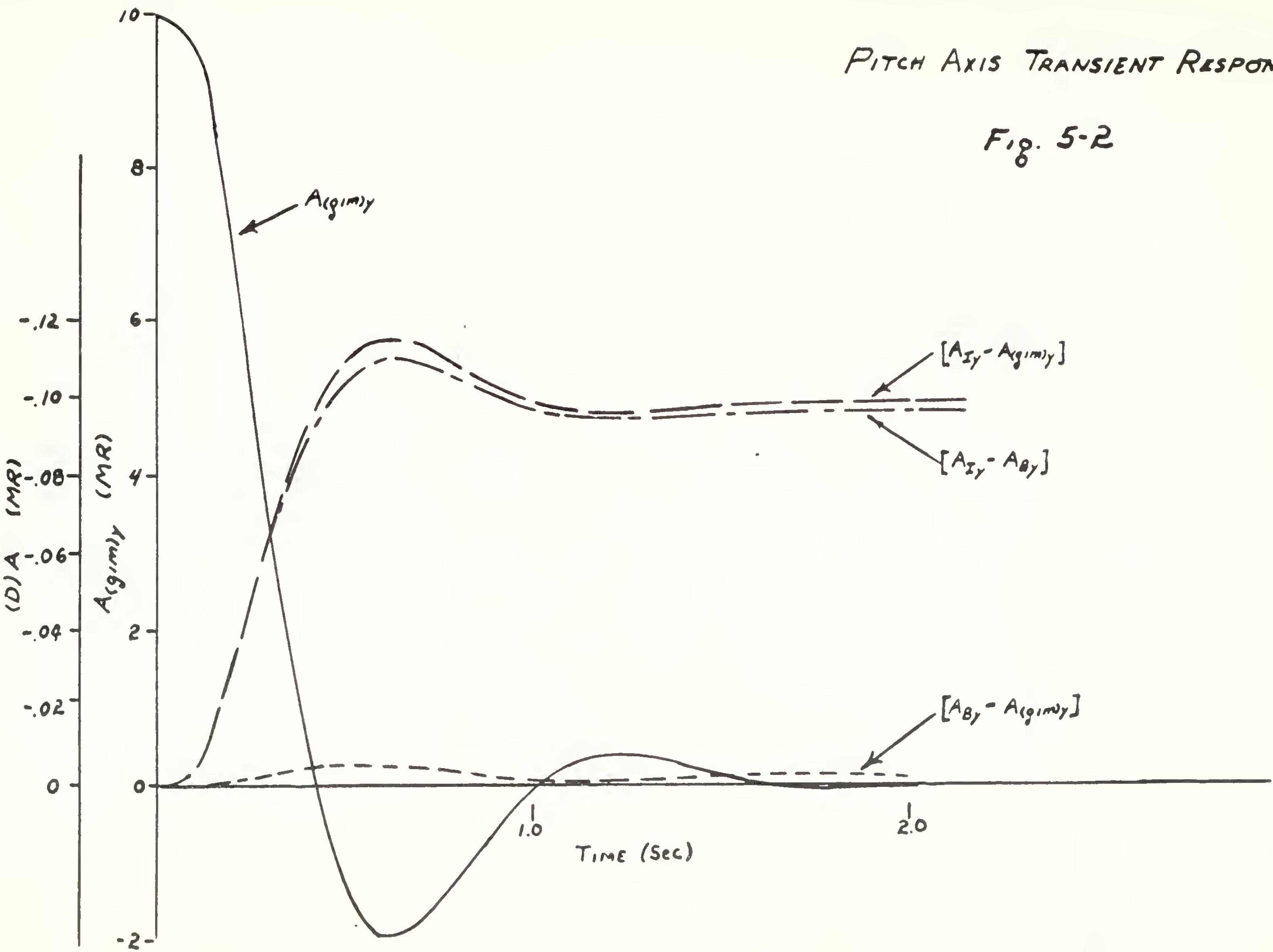
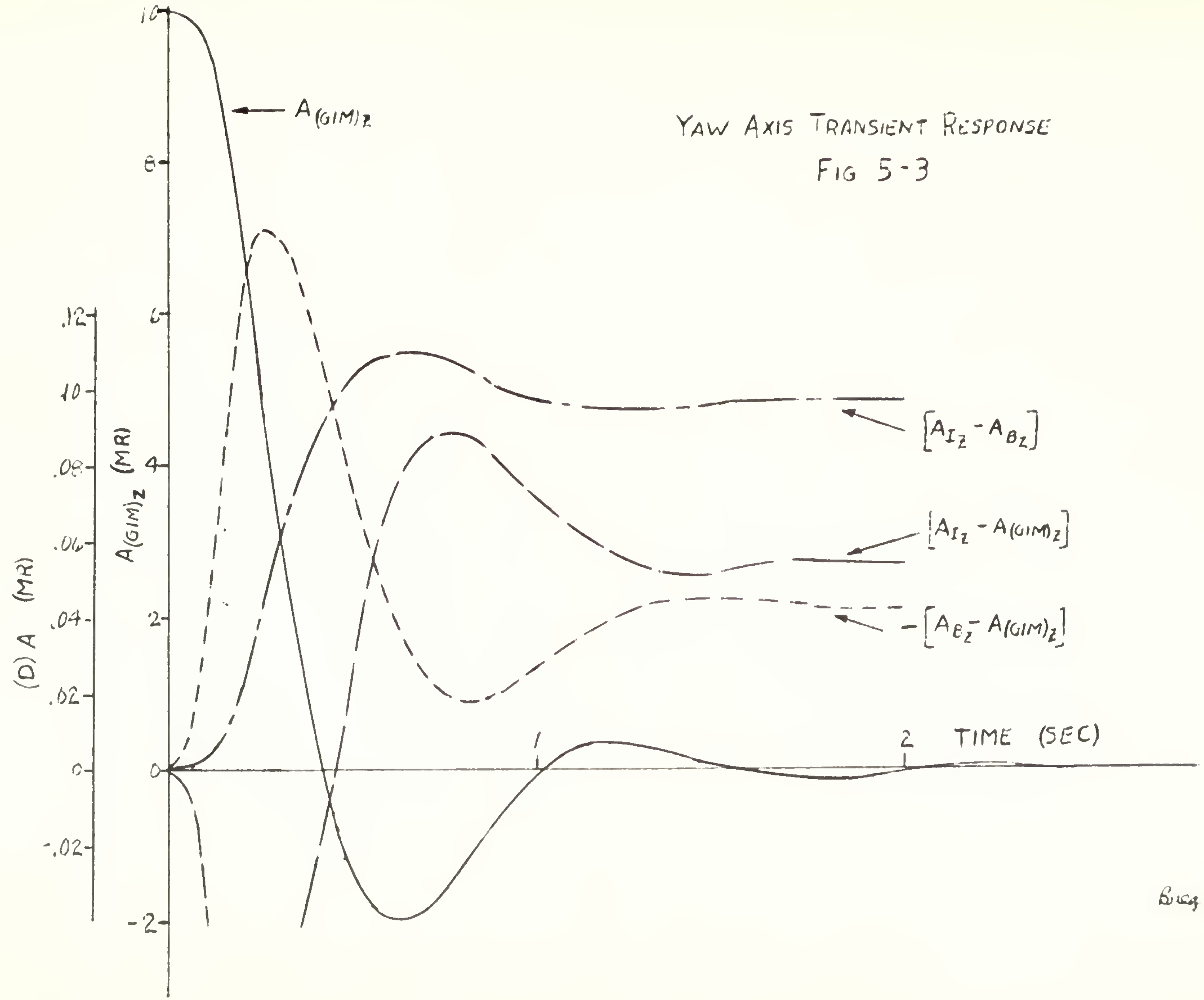


Pitch Axis Transient Response

Fig. 5-2



YAW AXIS TRANSIENT RESPONSE
FIG 5-3

for the pitch and yaw axes show a damped oscillatory response. The response in roll was only run until the gimbal angles on the other two axes were essentially zero. Stability of the roll axis system was not a problem since proportional control was used in the torque generator and the loop sensitivity was relatively low.

The root locus of the linearized pitch control system was discussed in Chapter III and shown in Fig. 3-3. The close loop gain gave a damping ratio of 0.445 for the two oscillatory modes of the system. The control system in pitch and yaw is a fifth order system in angular velocity as shown in Chapter III. The system is also non linear, but some feeling for system characteristics may be obtained by comparing it with a linear second order system. Chapter 19 of Ref. 6 contains curves of damping ratio versus transient peak ratio for linear second order systems. Entering these curves with the transient peak ratio obtained from the system response gives a damping ratio. A damped period of oscillation may also be obtained from the transient response. An undamped natural period may be calculated using the damping ratio.

The transient responses shown in Figs. 5-1 and 5-2 are not those of a second order system. Therefore values of damping ratio and undamped natural frequency based on second order response characteristics are meaningful only as a rough measure of the system characteristics. From Fig. 5-1, the yaw axis response indicates a damping ratio of 0.48 and an undamped natural frequency of 1.19 cycles per second. From the pitch axis response, Fig. 5-2, a damping ratio of 0.46 and an undamped natural frequency of 0.95 cycles per second were obtained. In Fig. 5-3 the slight recurvature of the gimbal angle curve indicates that the roll control system is rather heavily damped. The transient response

indicates that the system is dynamically stable on all axes.

The differences between inertial, body, and gimbal angles are plotted in Figs. 5-1, 5-2, and 5-3. No interfering angular accelerations were applied to the system during the transient response. The existence of these differences during the transient response indicates that any angular motion of the missile may cause errors in orientation. The absence of any means in the system of sensing these differences is shown by their constant value once the gyro gimbal angles have reached their steady state null position.

Information on the source of the differences can be obtained by considering the orientation of the gyro axes relative to the missile. To aid in comparison, the body axes orientation of the gyros are tabulated below.

	Gyro Axes		
Rotation Sensed	IA	SRA	OA
Roll	X	Y	Z
Pitch	Y	X	-Z
Yaw	Z	Y	-X

Table 5-1

INTEGRATING GYRO ORIENTATION

In the equations (2-10) for the integrating gyro, the terms introducing differences between body and gimbal angle are angular accelerations of the missile about the gyro output axis and the product of gyro gimbal angle and angular velocity of the missile about the gyro spin reference axis.

Comparing the body to gimbal angle difference curves for the roll and yaw axes shows a marked similarity in the oscillatory character of the differences. Both gyros are oriented with their spin reference

axes along the pitch (Y) axis of the missile. This would indicate that the dominant cause of this difference was the result of angular velocity about the spin reference axis. The greater magnitude of the error in the roll axis plot can be accounted for by the fact that the gimbal angle remains larger for the roll axis than for the yaw axis.

The relatively small effect on the body to gimbal angle difference resulting from angular accelerations about the output axis is indicated in the pitch axis plot. The pitch axis gyro has its spin reference axis along the roll axis. The slow rate of change of gimbal angle for the roll axis gyro shows that the roll rate of the missile is low. This indicates that the pitch gyro is little affected by angular rotation about its spin reference axis. The output axis of the pitch gyro is along the yaw (Z) axis. The curvature of the Z-axis gyro gimbal angle curve indicates the magnitude of the angular acceleration of the missile about that axis. It will be noted that the peaks of the oscillations in the pitch axis body to gimbal angle difference curve occurs when the Z-axis gimbal angle curve changes direction. In this transient response the angular acceleration of the body about a gyro output axis appears to result in little error. However, no general statement seems warranted for the case of arbitrary missile motion.

The inertial to body angle difference curves are almost identical for the pitch and yaw axes, while this difference is almost nonexistent in the roll axis plot. This difference is the result of finite rotations as was shown in Chapter II. For the roll axis the difference equation is:

(5-1)

$$A_{X_I} - A_{X_B} = \int A_{Y_I} W_{Z_B} dt - \int A_{Z_I} W_{Y_B} dt$$

For the other axes the difference is likewise the difference between the products of body angle and angular velocity components associated with the other two axes.

Comparison of Figs. 5-1 and 5-2 shows that the gimbal angle curves are almost identical. This means that at any time during the response the body angles and rotation rates are nearly identical for the Y and Z-axes. The two terms in the difference equation above therefore sum to almost zero. This also makes the pitch and yaw axes differences almost identical with time as shown in the figures.

The system response to an initial angular displacement could have a parallel in an actual missile. A similar event would occur in the case of a second or later stage of a multiple stage missile. In the interval between separation and ignition of the next stage, a transient motion of the missile may be expected. This transient motion imposes the initial condition on the control system when the thrust engine fires and control is commenced. If the angular velocities are small and can be neglected, this would establish initial conditions similar to those used for the transient response. The body would have a deviation in orientation with zero rotation rates. For the response calculated the difference between inertial angle and body angle was approximately one per cent of the initial displacement, for all three axes. This difference is the total orientation error developed during the response.

The information obtained from this calculation of the transient response of the system is as follows:

1. The control system is dynamically stable.
2. Rotations of the missile produce errors in orientation

due to the effects of three dimensional finite rotations and the characteristics of the integrating gyro.

3. These errors in orientation are not sensed by the system and exist in steady state.

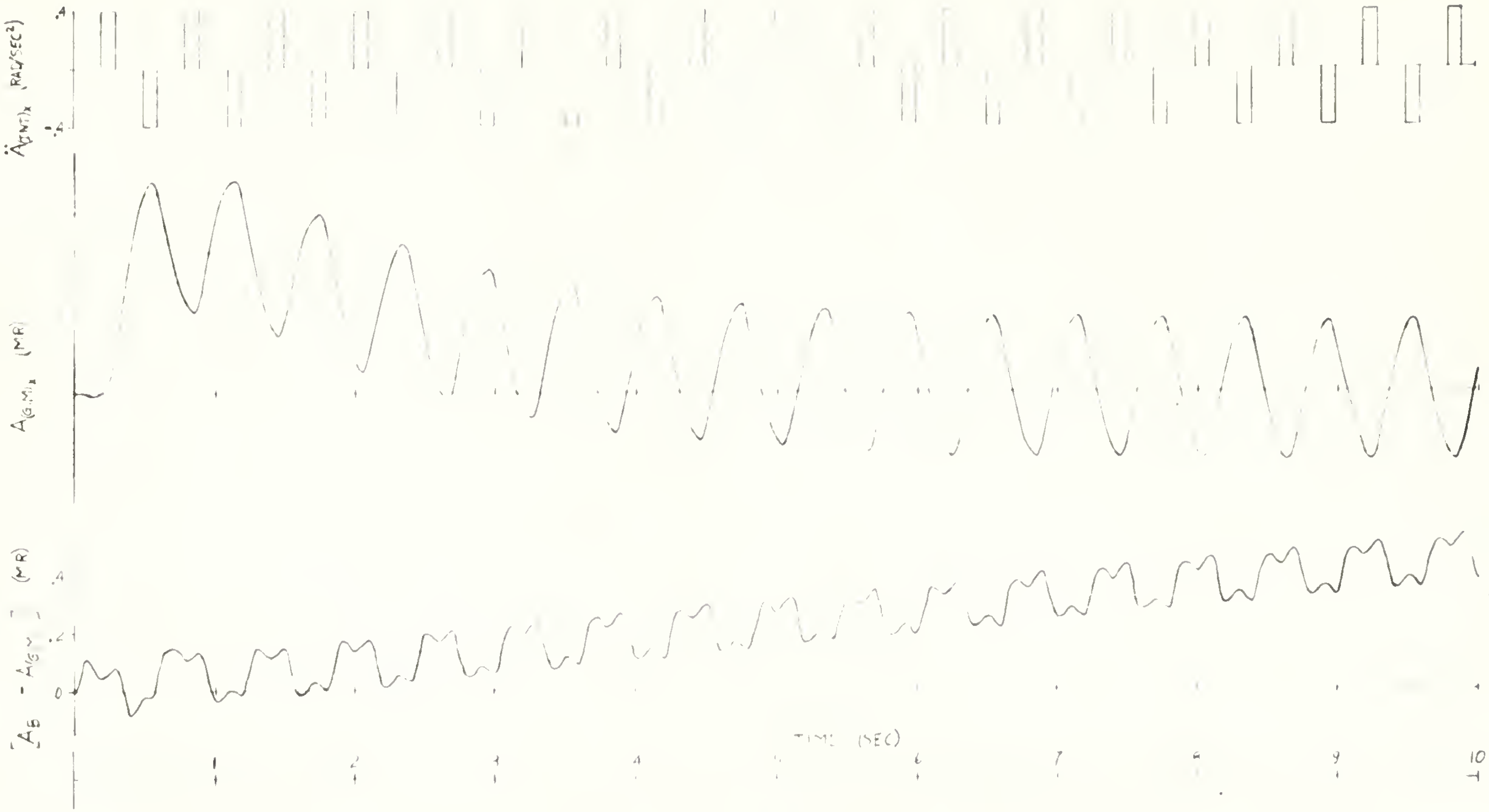
4. For the response computed the total steady state error in orientation was approximately one per cent of the initial displacement.

Periodic Forcing

The system response to a periodic forcing function in interfering angular acceleration was obtained. For reasons already discussed in Chapter IV all of the forcing functions used in this analysis were composed of combinations of square pulses. For the periodic forcing function the sequence of pulses shown in Figs. 5-4, 5-6, and 5-8 was arbitrarily selected. On a given axis of the system these pulses are alternately positive and negative with a period of six hundred milliseconds. Each pulse has a duration of one hundred milliseconds and a magnitude of four hundred milliradians per second squared. The phasing of the pulses on the various axes was arbitrarily chosen so that the yaw axis was two thirds of a period behind the pitch axis and the roll axis was two thirds of a period behind the yaw axis.

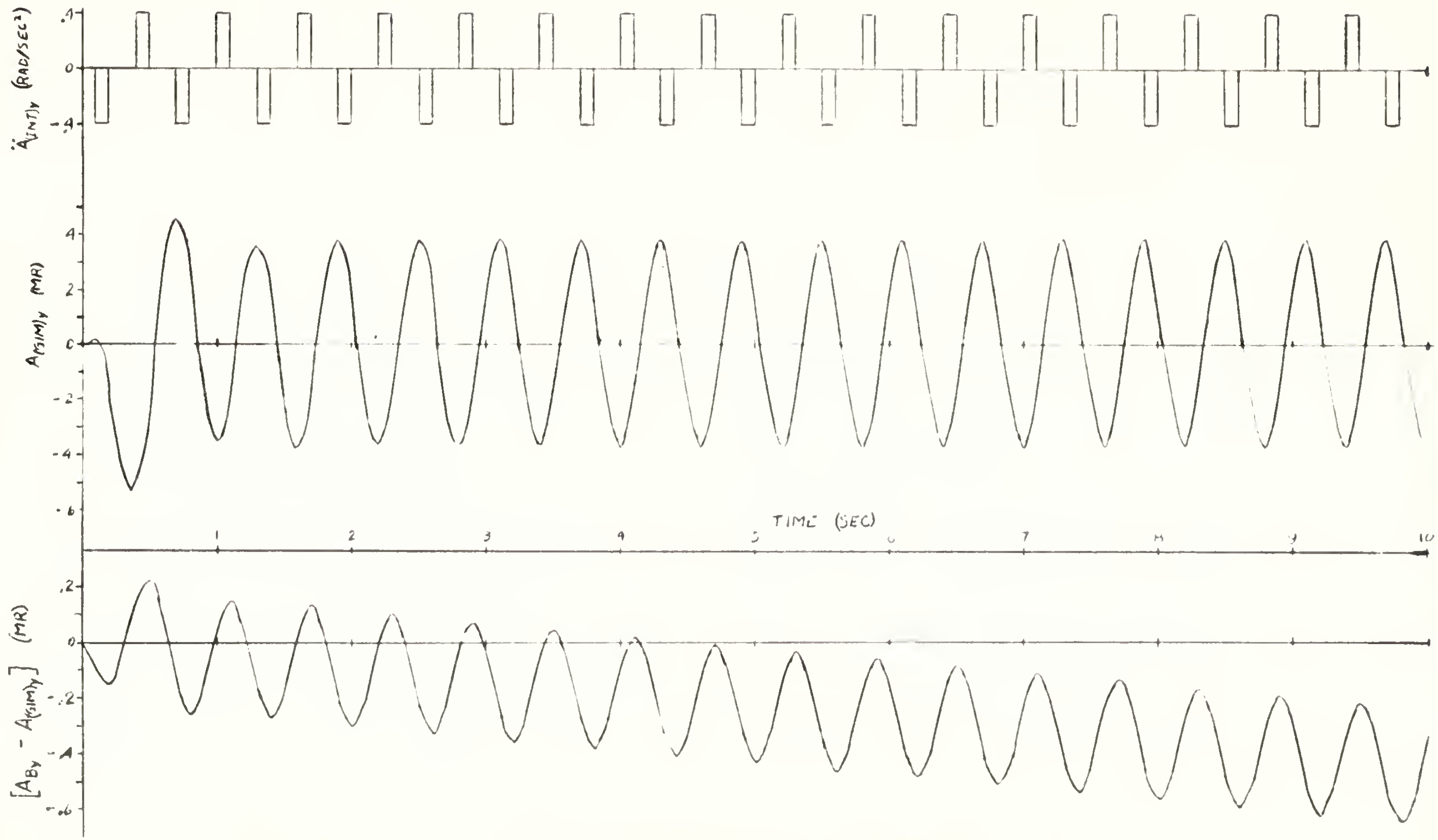
The periodic forcing function just described is equivalent to a linear vibrational acceleration of the missile of four feet per second squared at a distance of ten feet from the center of gravity. In the absence of system restoring torque this acceleration acting for the duration of the pulse would produce a displacement of about a quarter of an inch at ten feet.

The response of the system to the periodic forcing function just described is plotted in Figs. 5-4 through 5-9 for all axes. These



ROLL AXIS RESPONSE TO A PERIODIC PULSE FORCING FUNCTION
FIG. 5-4

ROLL AXIS RESPONSE TO A PERIODIC PULSE FORCING FUNCTION
ORIENTATION ERROR
FIG 5



PITCH AXIS RESPONSE TO A PERIODIC PULSE FORCING FUNCTION

FIG. 5-6

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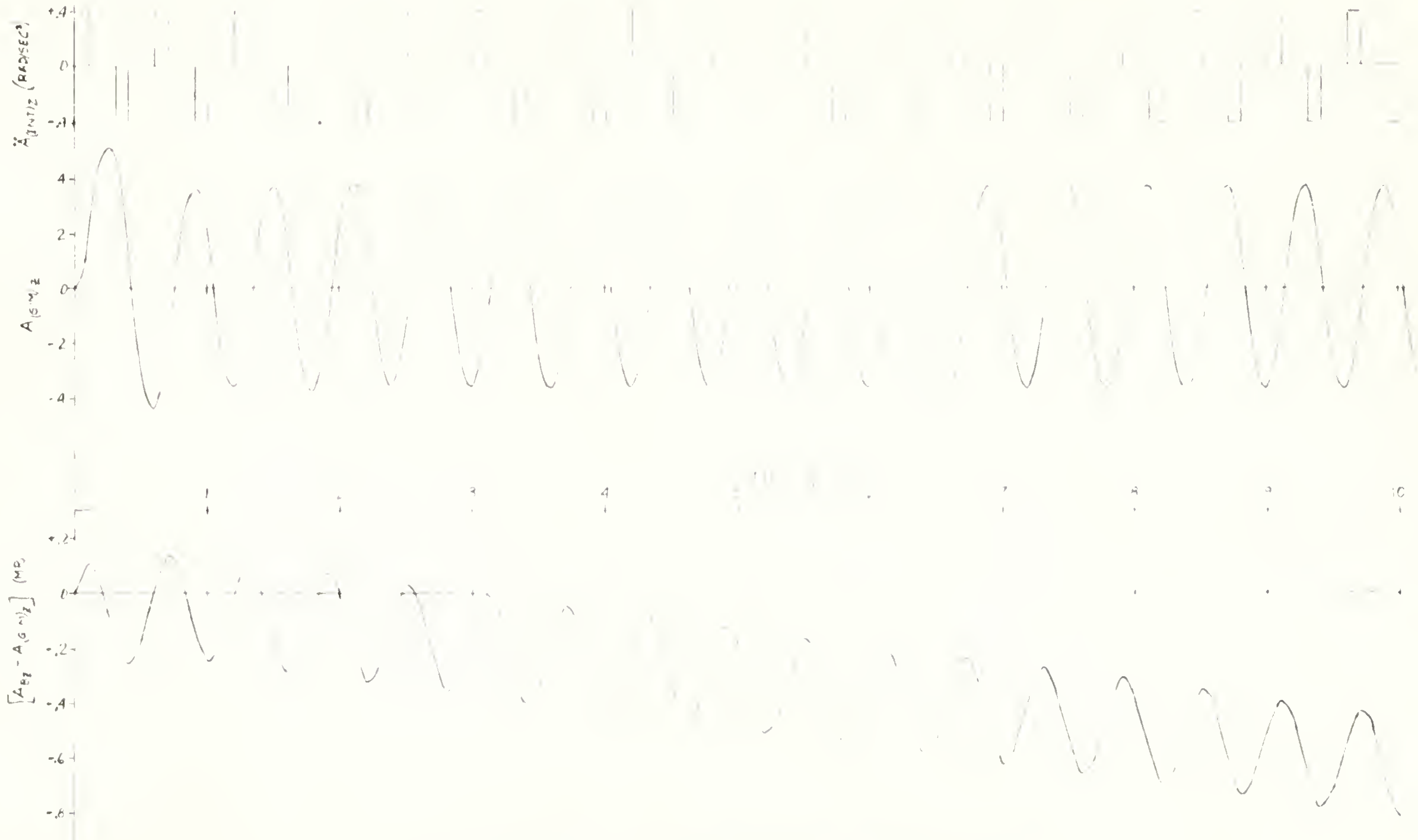
Pitch Axis Response To A Periodic [illegible] Forcing Function
Orientation Errors
Fig 5-7



YAW AXIS RESPONSE TO A PER[illegible] FORCING FUNCTION

FIG. [illegible]

YAW AXIS RESPONSE TO A PERIODIC PULSE FORCING FUNCTION
ORIENTATION ERROR
FIG. 5.4

plots show the forcing function, the gyro gimbal angle and the three angular differences for each axis for the first ten seconds of system operation. At the end of ten seconds the system had reached steady state on all axes so that further computation was unnecessary.

The plot of the gyro gimbal angle is essentially the same for all axes. After an initial transient period the gimbal angle settled down to a steady oscillation about its null position which appears to be almost sinusoidal. However, comparison of these plots with those of Figs. 5-1 through 5-3 shows that the gimbal angle response is actually made up of a continuous series of exponential curves similar to those obtained for the transient response.

The behavior of the gimbal angle during the initial transient period is consistent with the results of the transient analysis discussed previously. Each axis develops some dynamic error during the transient period which is eventually damped out. In the case of the pitch and yaw axes this damping requires approximately two seconds while for the roll axis approximately six seconds are required.

The effects of coupling among axes in the system can be seen in the plots of gimbal angle for the pitch and roll axes. Each of these axes is initially unforced and yet in each case some small gimbal angle develops immediately due to the motion of the missile about the yaw axis.

The angular differences of Fig. 5-4 through 5-9 all develop in a fairly consistent pattern in spite of the variation in dynamics between axes of the system. Some general discussion of the basic patterns is warranted before considering the results axis by axis.

The difference between body angle and gyro gimbal angle is, in each case, an oscillatory curve which slowly drifts away from zero.

In steady state this is a combination of a steady oscillation about zero and a constant drift rate. This can be explained in terms of the two error producing effects discussed in connection with the integrating gyro.

In steady state the components of angular acceleration of the missile are symmetric about zero in the present case. Therefore for a full cycle the net angular acceleration of the missile about a given axis is zero. The difference between body and gimbal angle may oscillate due to the effect of angular acceleration about a gyro output axis but for a full cycle the net difference produced by its effect is zero. Any drift in the plot of body to gimbal angle difference must then be due to the effect of angular velocity of the missile about the gyro spin reference axis.

The product of gyro gimbal angle and missile angular velocity about the gyro spin reference axis will not in general integrate to zero over a full cycle. Therefore over a full cycle a net difference between body and gyro gimbal angles will usually exist. In steady state the difference due to this effect will be the same for each cycle. This produces the linear increase upon which the oscillatory effects are superimposed.

The curves for the difference between inertial and body angles are characterized by a small oscillation, the mean of which is linear with a finite slope in steady state. This linear divergance is to be expected for the case of periodic forcing. A net difference developed during a given cycle of the forcing function would also be developed in any other cycle in steady state. Therefore there would be a linear accumulation of these differences with time such as is shown in the curves.

The sum of the differences just discussed is the difference between the inertial angle and the gyro gimbal angle. This difference is also plotted for each axis. It represents the total error between the reference orientation and the orientation to which the system is currently attempting to drive the missile due to all of the effects under investigation.

The angular differences for each axis at the end of ten seconds are given in Table 5-2. Since all differences steadily increase in magnitude these are the maximum errors in orientation which the system experiences for the ten second run. For those differences which are oscillatory the numbers given are the mean error at ten seconds.

Table 5-2 also gives the ratio of the angular differences to the maximum amplitude of the missile angular oscillation in steady state. Since the gyro sensitivity from body angle to gyro gimbal angle is unity, the maximum amplitude of gyro gimbal angle oscillation can be used in forming ratios. However, it is more meaningful to relate errors in orientation to body angular motion. It should be noted that the magnitude of the body angle is not that of the gimbal angle due to the errors. However, the amplitude of the body angle oscillations are the same as the amplitude of the gimbal angle oscillations.

The ten seconds of running time for which the results of periodic forcing have been determined is short compared to the firing time of most missiles. A good approximation to missile firing time would be one hundred seconds which would require a computer run ten times as long as the one that was made. However, once the system has reached steady state operation and remained there long enough to establish the trend of the errors further computation is unnecessary. The error

		AXIS	ROLL	PITCH	YAW
$A_I - A_B$	10 Sec.	MAG. (MR)	1.28	0.58	0.87
		$\measuredangle A_{gim_{max}}$	53	15	23.5
	100 Sec.	MAG. (MR)	12.8	5.8	8.7
		MAG. (DEG)	0.734	0.332	0.498
		$\measuredangle A_{gim_{max}}$	530	150	235
$A_B - A_{GIM}$	10 Sec.	MAG. (MR)	0.45	0.45	0.64
		$\measuredangle A_{gim_{max}}$	18.7	11.7	17.3
	100 Sec.	MAG. (MR)	4.5	4.5	6.4
		MAG. (DEG)	0.278	0.278	0.366
		$\measuredangle A_{gim_{max}}$	187	117	173
$A_I - A_{GIM}$	10 Sec.	MAG. (MR)	0.81	1.05	1.49
		$\measuredangle A_{gim_{max}}$	33.8	27.2	40
	100 Sec.	MAG. (MR)	8.1	10.5	14.9
		MAG. (DEG)	0.464	0.601	0.854
		$\measuredangle A_{gim_{max}}$	338	272	400
DIFF.	TIME	$A_{gim_{max}}$ (MR)	2.4	3.85	3.7

ORIENTATION ERRORS WITH PERIODIC FORCING

TABLE 5-2

curves, being linear, may be extrapolated to any desired value. In the present case this extrapolation has been carried out and the resulting errors in orientation are shown in Table 5-2 along with the values for ten seconds. The values of the differences at one hundred seconds are given in both radians and degrees for convenience.

The following information was obtained from the response of the system to a periodic interfering angular acceleration:

1. Errors in orientation developed due to the effects of three dimensional finite rotations and the characteristics of the integrating gyro.

2. These errors were systematic, and in steady state, their mean increased linearly with time.

3. The total error in orientation developed at the end of 10 seconds was 0.91 milliradians in roll, 1.05 milliradians in pitch, and 1.49 milliradians in yaw. These are respectively 34%, 27%, and 40% of the amplitude of missile angular motion about the axis in question.

4. Since the increase in error was linear, extrapolation to 100 seconds was possible. This increases the values above by a factor of ten.

<u>Random Forcing</u>

In order to simulate interfering angular accelerations as realistically as possible within the limitations already imposed by computing time, the response of the system to a series of random pulses was determined. Since no steady state could be expected for such a forcing function the problem was run for forty seconds of real time. This required approximately ten hours of computing time.

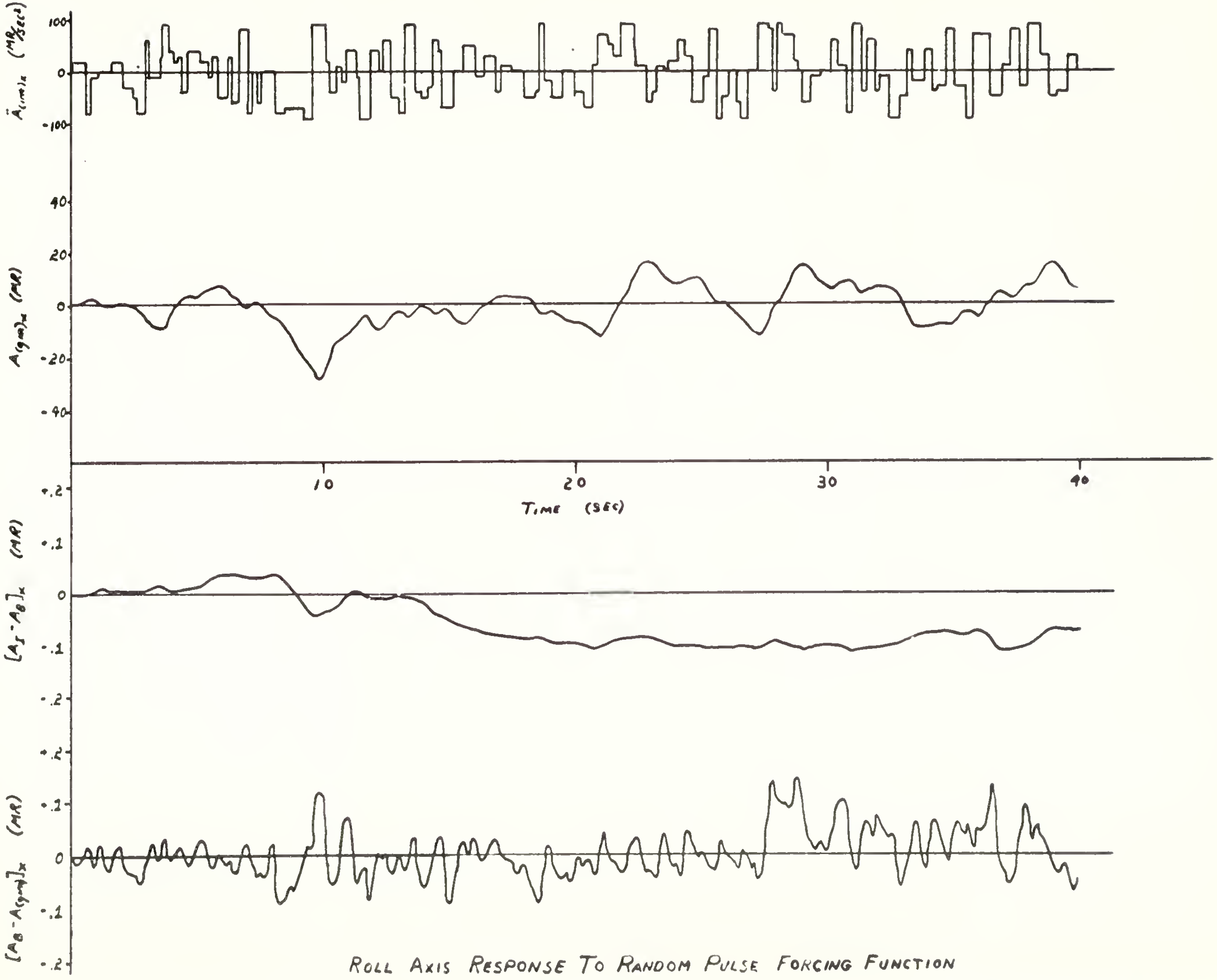
The random forcing function was made random with respect to

three of the pulse characteristics. The sign of the pulse, its magnitude, and its duration were independently chosen in a random fashion. This was done automatically in the computer. The sign of the pulse was chosen first. The mechanism by which this was done was such that positive and negative signs each had a probability of occurrence of one half in a given selection. The magnitude of the pulses was limited to the values from 0 to 90 milliradians per second squared, in steps of ten milliradians per second squared. Each value had a probability of occurrence of one tenth in a given selection. The duration of the pulses was selected at random in the range from 5 to 500 milliseconds with selection being limited to multiples of five milliseconds. The forcing function obtained in this manner is shown for each axis in Figs. 5-10 through 5-12. It should be noted that there is no correlation between the forcing functions for the various axes since selection on each axis was completly independent of the other two.

The response of the system to the random forcing function just described is shown in Figs. 5-10 through 5-13.

The system response as shown by gyro gimbal angle is about what could be expected in view of the previous results. The gimbal angle oscillates randomly in essentially the same manner as the forcing function. There is no steady state condition apparent in the response. None was expected since the forcing function is random. In general the behavior of the gimbal angle is consistent with the previous observations on system dynamic stability.

The angular differences have the same general character as those observed in the periodic case. The body to gimbal angle difference



ROLL AXIS RESPONSE TO RANDOM PULSE FORCING FUNCTION

Fig 5-10

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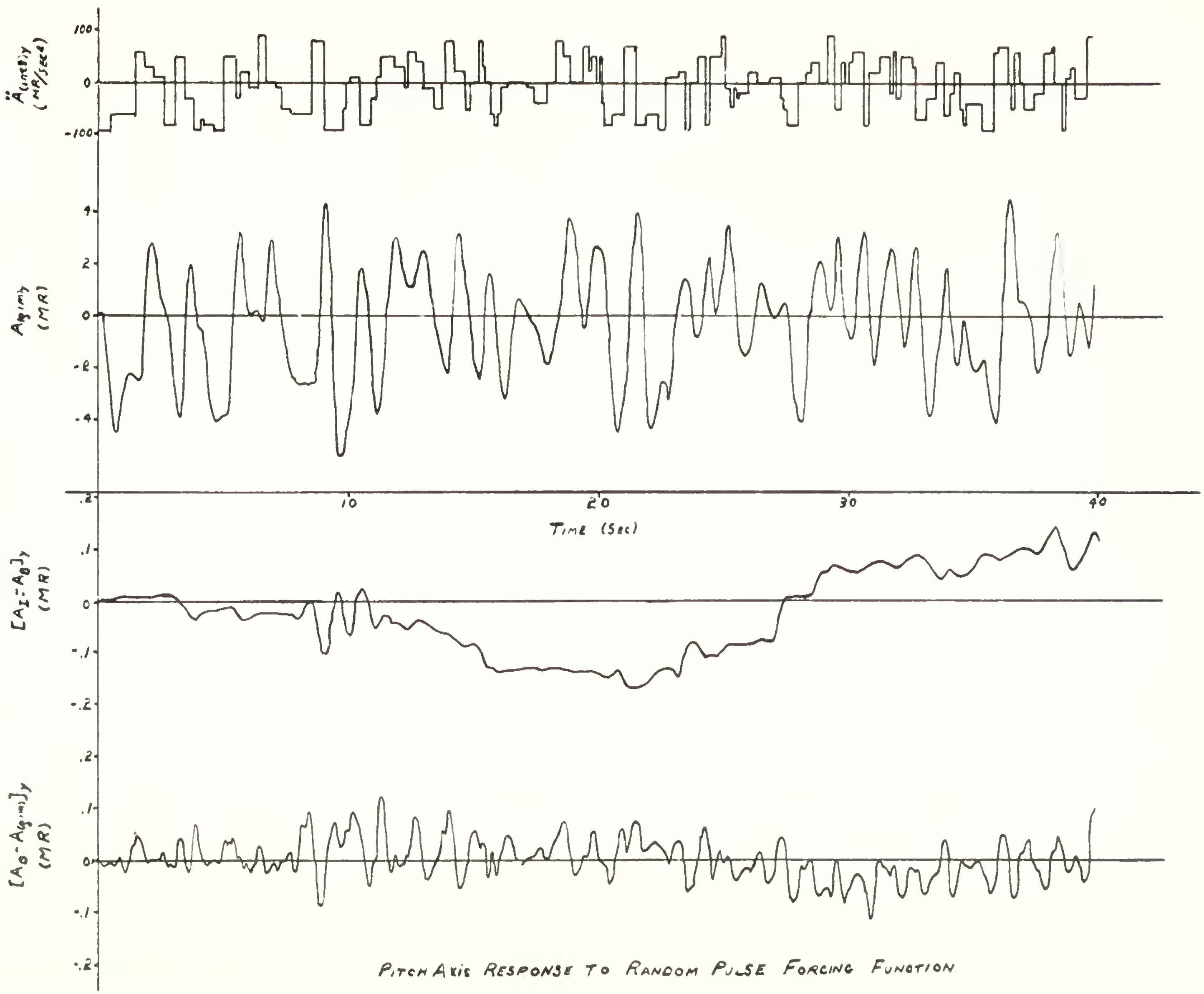

PITCH AXIS RESPONSE TO RANDOM PULSE FORCING FUNCTION

Fig 5-11

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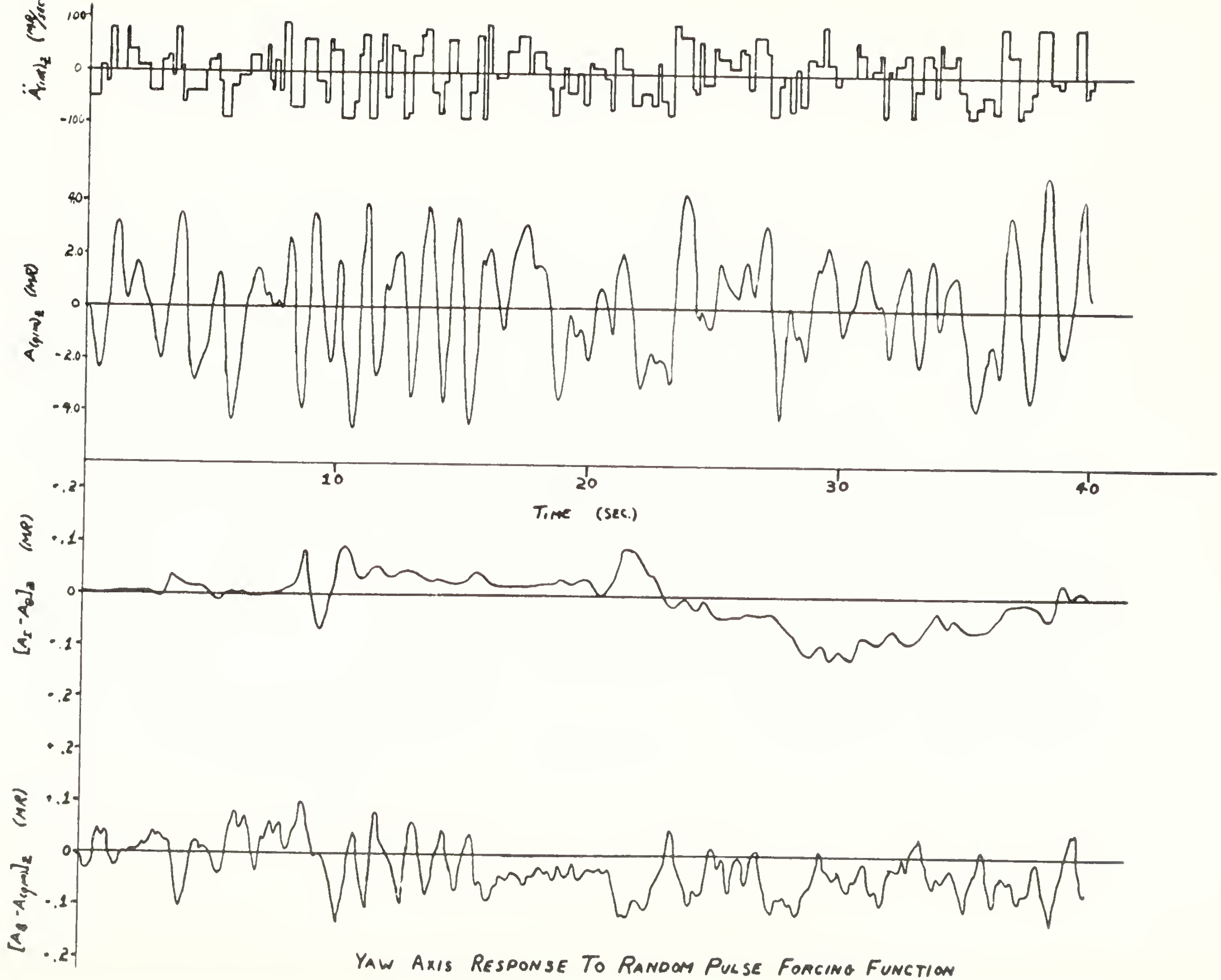

YAW AXIS RESPONSE TO RANDOM PULSE FORCING FUNCTION

Fig 5-12

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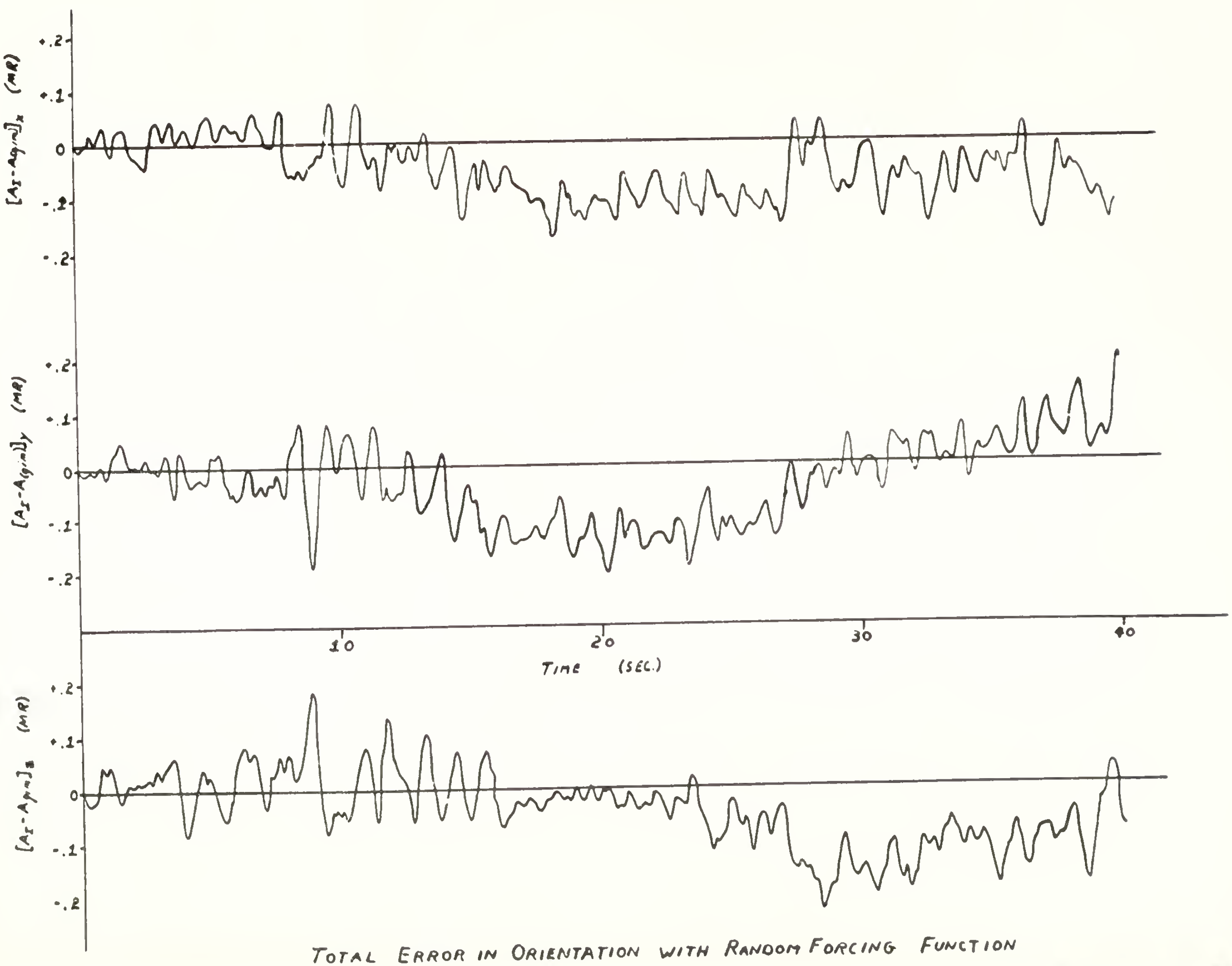

TOTAL ERROR IN ORIENTATION WITH RANDOM FORCING FUNCTION

Fig 5-13

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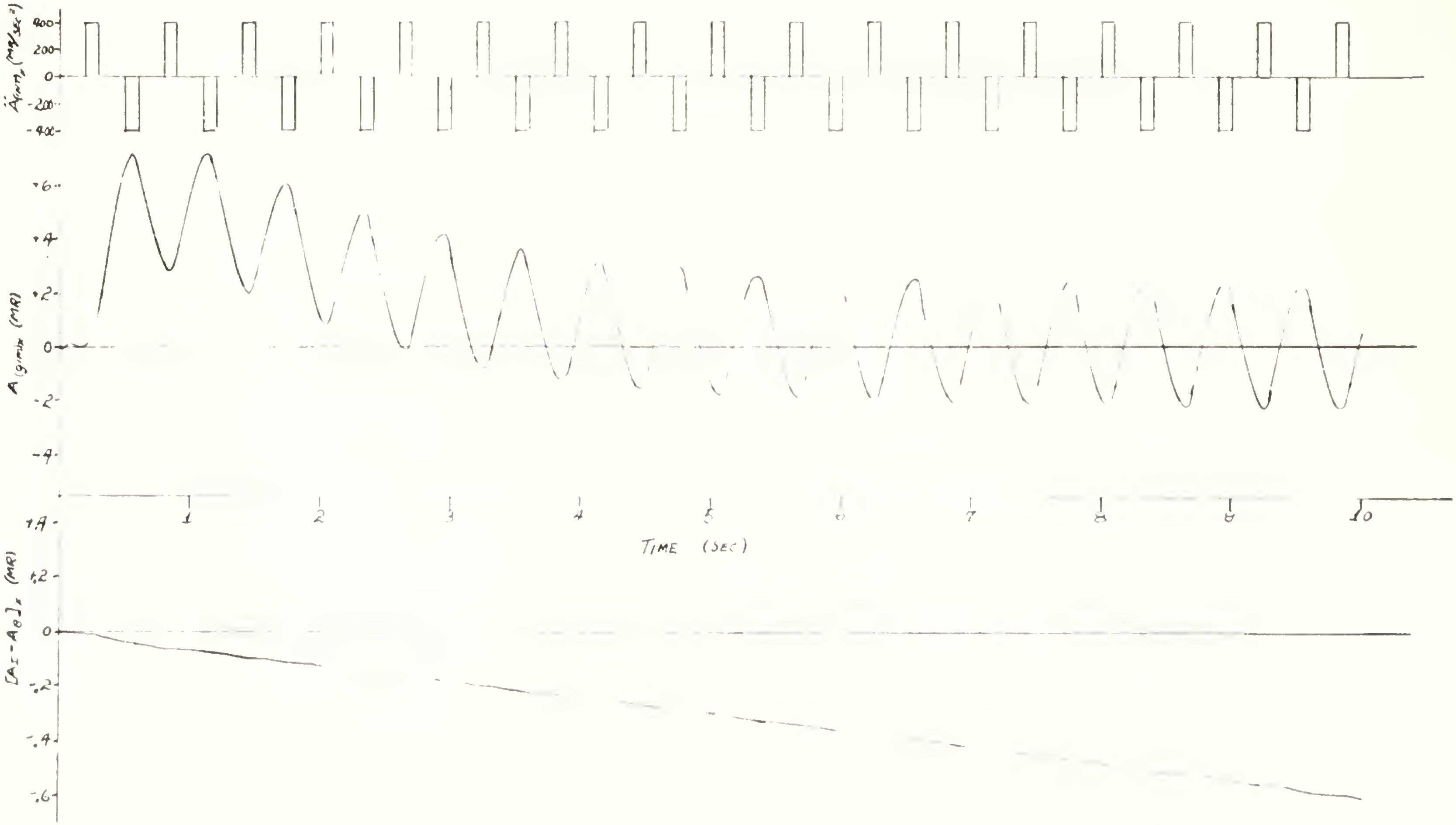
curve is oscillatory, while the inertial to body angle difference is relatively smooth. However, there is no systematic divergence of these angular differences for this run. In this respect the response was different from the results for periodic forcing.

Since the response to random forcing is essentially statistical in nature generalized statements can not be made on the basis of a single run. The angular difference curves shown are just one set of an infinite number that could occur as a result of this type of forcing. The fact that the angular differences obtained during one run do not diverge, does not imply that they would not diverge in another run. However, the single run shown does demonstrate that errors in orientation do exist even with random forcing. The interfering angular acceleration inputs need not be periodic for these orientational errors to develop; and these are the only conclusions that can reasonably be drawn from the results of a single run. More runs would permit a statistical approach to the problem. More generalized results could then be obtained.

A run was made to show that orientational errors will exist regardless of control system dynamics. The system dynamics were altered by assuming proportional response in the integrating gyros and the torque generating systems. The reduced equations are shown in Appendix B.

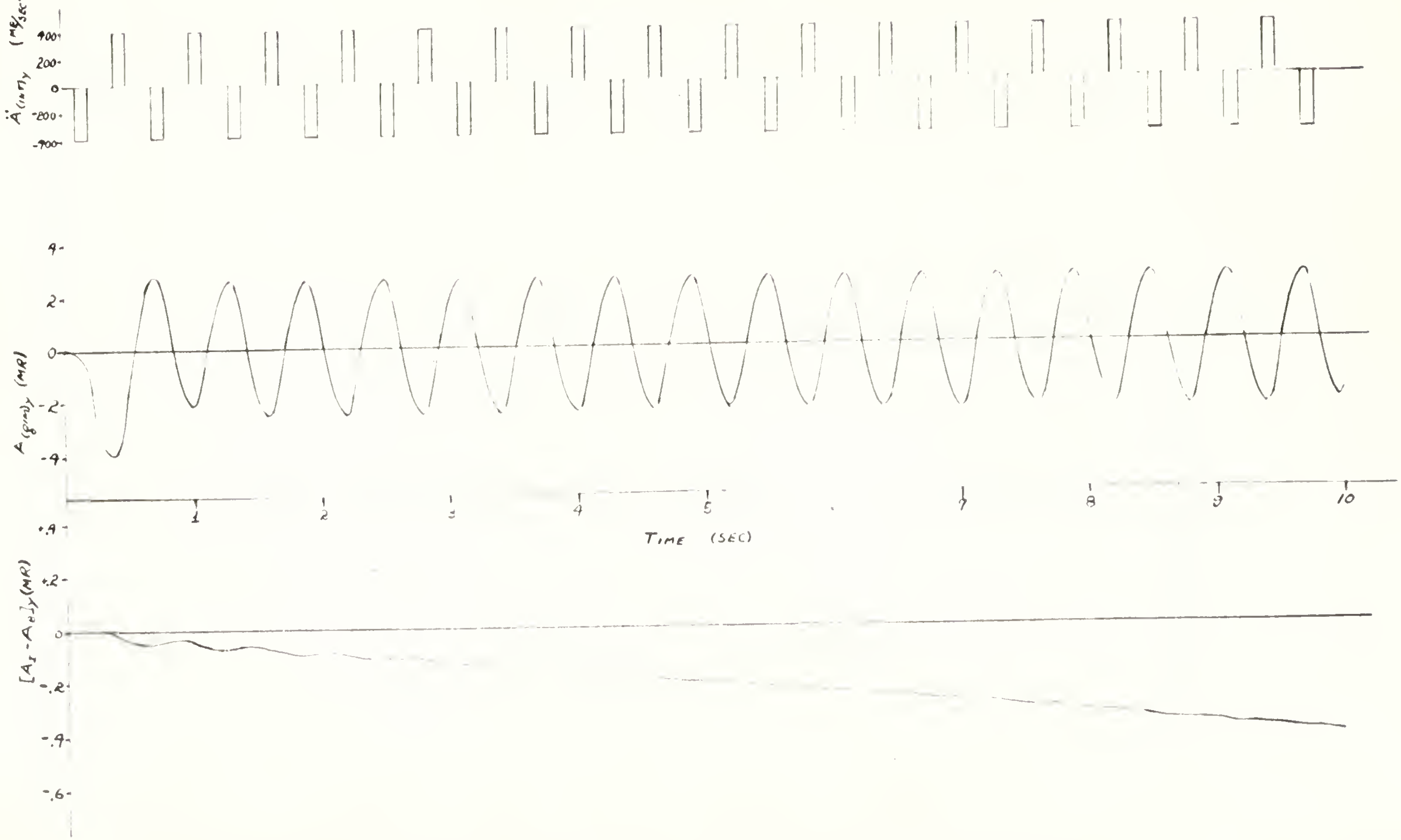
This simplified system was forced by the same periodic forcing function used previously. The response to this forcing is shown in Figs. 5-14 through 5-16.

It will be noted that the plots of this response show missile body angle as the reference variable. For this simplified system



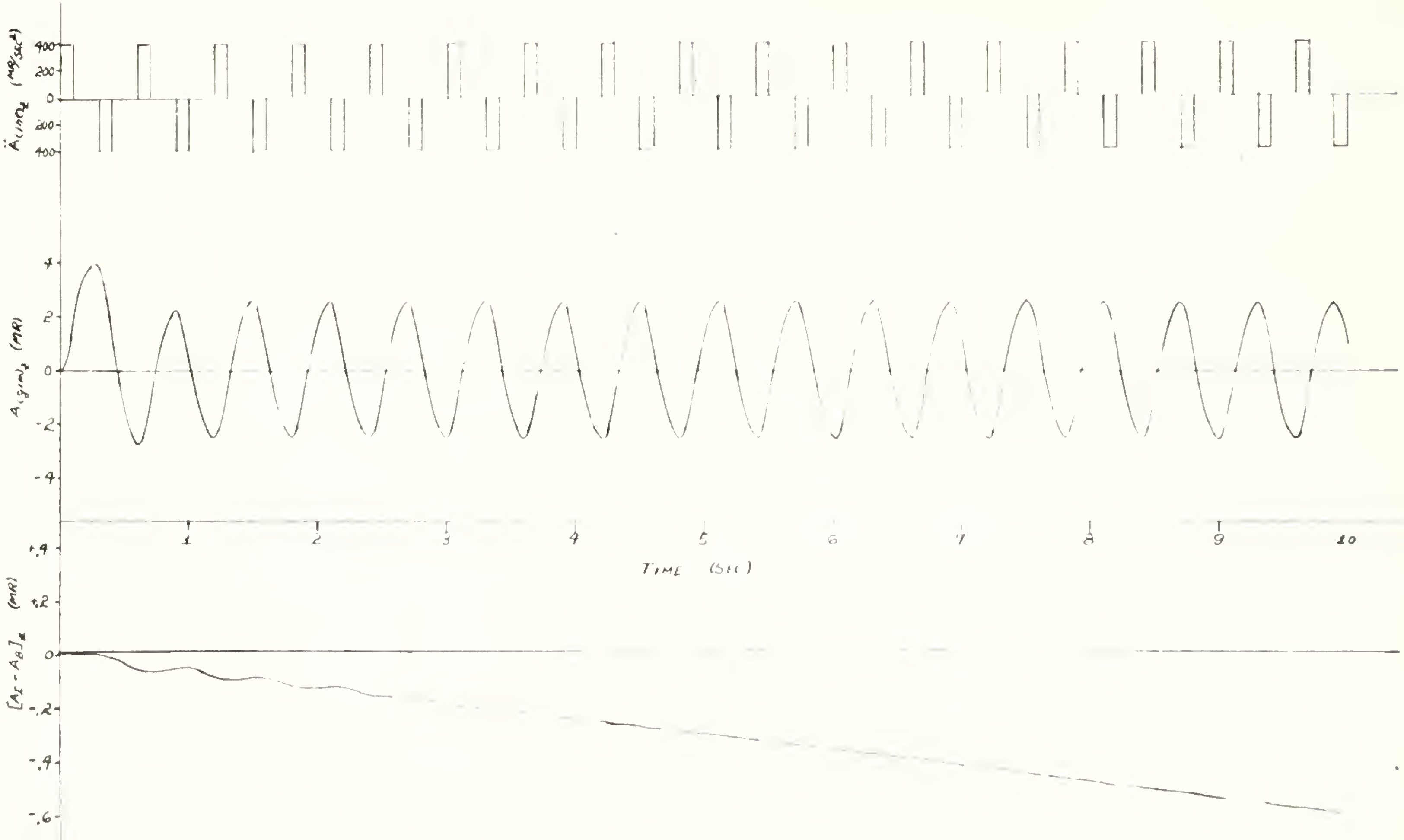
RESPONSE IN ROLL FOR SYSTEM WITH REDUCED DYNAMICS

Fig 5-14



RESPONSE IN PITCH FOR SYSTEM WITH REDUCED DYNAMICS

Fig 5-15



RESPONSE IN YAW FOR SYSTEM WITH REDUCED DYNAMICS

Fig 5-16

the gyro angle is identical to the body angle. Therefore the only difference that can occur is that between inertial and body angles. The angular difference curves from inertial to body angle indicate the error in orientation develops in the same manner as for the complete system. The error is slightly less in this case since the control system response is faster. Therefore the missile will experience smaller angular deviations and lower angular velocities for the same forcing function. This accounts for the reduction in the net error in orientation due to finite rotation effects. In any case the error does exist regardless of control system dynamics.

The results obtained in this investigation prove the existence of errors in orientation due to three dimensional finite rotations and integrating gyro characteristics. Some information was obtained for the cases of periodic forcing, random forcing, and transient response from an initial angular displacement.

For the case of periodic forcing, further investigation is needed to determine the effects of the frequency and magnitude of the forcing function. In the case of random forcing more runs will be required to permit a statical evaluation of the errors developed. If very many runs are to be made, a faster mehtod of computation is highly desirable. This can be accomplished through the use of a faster computer, or perhaps by changing the method of computation.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The preceding analysis has shown that errors in orientation may develop in the geometrical stabilization system using body fixed gyros. These errors are distinct from the deviations in orientation which such a system would experience due to system dynamic lags. It was demonstrated that under certain conditions the errors in orientation systematically increase with time. Also, it was shown that errors in orientation develop regardless of system dynamics.

Errors in orientation developed in the absence of interfering angular vibrations. In restoring from an initial angular displacement the system developed an error of about one per cent of the initial displacement. This error remained when the system had come to rest. Those errors developed due to the coupling of missile angular motion between axes of the system.

When the system was forced with a periodic interfering angular acceleration on all axes, there was a systematic increase in orientation errors with time. After ten seconds the total error in orientation in yaw was forty per cent of the maximum amplitude of the angular oscillation of the missile in steady state. Corresponding figures for the pitch and roll axes were twenty seven per cent and thirty four per cent respectively. These errors increase linearly with time. Therefore at one hundred seconds the error in yaw becomes four hundred per cent of

the maximum amplitude of the angular oscillation of the missile.

The same kind of errors in orientation developed when the interfering angular accelerations were random. However, due to the statistical nature of the forcing function no conclusions could be made on the basis of a single run concerning the manner in which the errors will accumulate with time.

Further investigation of the problem is still required. The investigation of periodic forcing should be extended to determine the effects of the frequency and magnitude of the interfering angular accelerations. Enough results should be obtained using a random forcing function to permit a statistical analysis of the errors. These additional investigations would be greatly facilitated if a significant reduction in computing time could be achieved.

APPENDIX A

THE TRANSFER OF VECTOR COMPONENTS BETWEEN ROTATED AXES SETS

Fig. A-1 shows the inertial and body axes sets described in Chapter II and an arbitrary vector $\overline{V}$ which is fixed in the body axes set. The problem is to express the components of the vector in the inertial axes set when the body axes components are known and the body axes have assumed an arbitrary orientation. In general, an arbitrary orientation of the body axes relative to the inertial axes can be duplicated by, at most, three successive finite rotations. In the analysis below three successive rotations of the body, one about each of the inertial axes, are assumed. After each rotation the components of the vector $\overline{V}$ in inertial axes are derived. After three rotations the results for the individual rotations are combined to give the components of the vector in inertial axes in terms of the three angles of rotation and the components in body axes. The resulting transformation equation is not unique. There is a different transformation for each possible order of rotations. However, a simplification is made at the end which gets around this difficulty. In the derivation below the order of rotations is about the X, Y, and Z-axes in that order.

Let the body axes of Fig. A-1 be rotated about the X inertial axis through the angle A_{X_I} as shown in Fig. A-2. From the figure it

can be seen that the components of the vector $\overline{V}$ in inertial axes can now be written:

(A-1)

$$V_{x_I} = V_{x_B}$$

$$V_{y_I} = V_{y_B} \cos A_{x_I} - V_{z_B} \sin A_{x_I}$$

$$V_{z_I} = V_{y_B} \sin A_{x_I} + V_{z_B} \cos A_{x_I}$$

There is considerable simplification afforded by writing the set of equations (A-1) in matrix form.

(A-2)

$$\begin{bmatrix} V_{x_I} \\ V_{y_I} \\ V_{z_I} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & CA_{x_I} & -SA_{x_I} \\ 0 & SA_{x_I} & CA_{x_I} \end{bmatrix} \begin{bmatrix} V_{x_B} \\ V_{y_B} \\ V_{z_B} \end{bmatrix}$$

Where the notation SA_{x_I}, and CA_{x_I} refers to the sine and cosine of the angle of rotation about the X-axis.

Now let a new set of body axes (B') be defined such that they are coincident with the inertial axes of Fig. A-2. Note that the transformation (A-2) always applies between the sets of body axes (B) and (B'). That is:

(A-3)

$$\begin{bmatrix} V'_{x_B} \\ V'_{y_B} \\ V'_{z_B} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & CA_{x_I} & -SA_{x_I} \\ 0 & SA_{x_I} & CA_{x_I} \end{bmatrix} \begin{bmatrix} V_{x_B} \\ V_{y_B} \\ V_{z_B} \end{bmatrix}$$

If the body is now rotated about the Y inertial axis through

A_{y_I} then from Fig. A-3 it can be seen that:

(A-4)

$$\begin{bmatrix} V_{x_I} \\ V_{y_I} \\ V_{z_I} \end{bmatrix} = \begin{bmatrix} CA_{y_I} & 0 & SA_{y_I} \\ 0 & 1 & 0 \\ -SA_{y_I} & 0 & CA_{y_I} \end{bmatrix} \begin{bmatrix} V'_{x_B} \\ V'_{y_B} \\ V'_{z_B} \end{bmatrix}$$

and defining the body axes set (B'') in exactly the same manner as (B') gives:

(A-5)

$$\begin{bmatrix} V''_{x_B} \\ V''_{y_B} \\ V''_{z_B} \end{bmatrix} = \begin{bmatrix} CA_{y_I} & 0 & SA_{y_I} \\ 0 & 1 & 0 \\ -SA_{y_I} & 0 & CA_{y_I} \end{bmatrix} \begin{bmatrix} V'_{x_B} \\ V'_{y_B} \\ V'_{z_B} \end{bmatrix}$$

Rotating now about Z_I through A_{z_I} gives:

(A-6)

$$\begin{bmatrix} V_{x_I} \\ V_{y_I} \\ V_{z_I} \end{bmatrix} = \begin{bmatrix} CA_{z_I} & -SA_{z_I} & 0 \\ SA_{z_I} & CA_{z_I} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V''_{x_B} \\ V''_{y_B} \\ V''_{z_B} \end{bmatrix}$$

Replacing the matrices above by symbols of the form (V_I) allows equation (A-6) to be written:

(A-6)

$$(V_I) = (A_z)\,(V''_B)$$

where the meaning of the notation becomes evident by comparing the two forms of the equation (A-6).

If (A-3) is substituted into (A-5) and this equation in turn is

substituted into (A-6) the result is:

(A-7)

$$(V_I) = (A_z)\,(A_y)\,(A_x)\,(V_B)$$

If the matrix multiplication $(A_x)\,(A_y)\,(A_z)$ is carried out the resulting equation for the inertial components in terms of body components becomes:

(A-8)

$$\begin{bmatrix} V_{x_I} \\ V_{y_I} \\ V_{z_I} \end{bmatrix} = \begin{bmatrix} CA_{z_I}CA_{y_I} & CA_{z_I}SA_{y_I}SA_{x_I} - SA_{z_I}CA_{x_I} & CA_{z_I}SA_{y_I}CA_{x_I} + SA_{z_I}SA_{x_I} \\ SA_{z_I}CA_{y_I} & SA_{y_I}SA_{x_I}SA_{z_I} + CA_{z_I}CA_{x_I} & SA_{z_I}SA_{y_I}CA_{x_I} - CA_{z_I}SA_{x_I} \\ -SA_{y_I} & CA_{y_I}SA_{x_I} & CA_{y_I}CA_{x_I} \end{bmatrix} \begin{bmatrix} V_{x_B} \\ V_{y_B} \\ V_{z_B} \end{bmatrix}$$

If the rotations about the inertial axes are kept small, so that the product of the sine of two of the angles can be neglected this can be written:

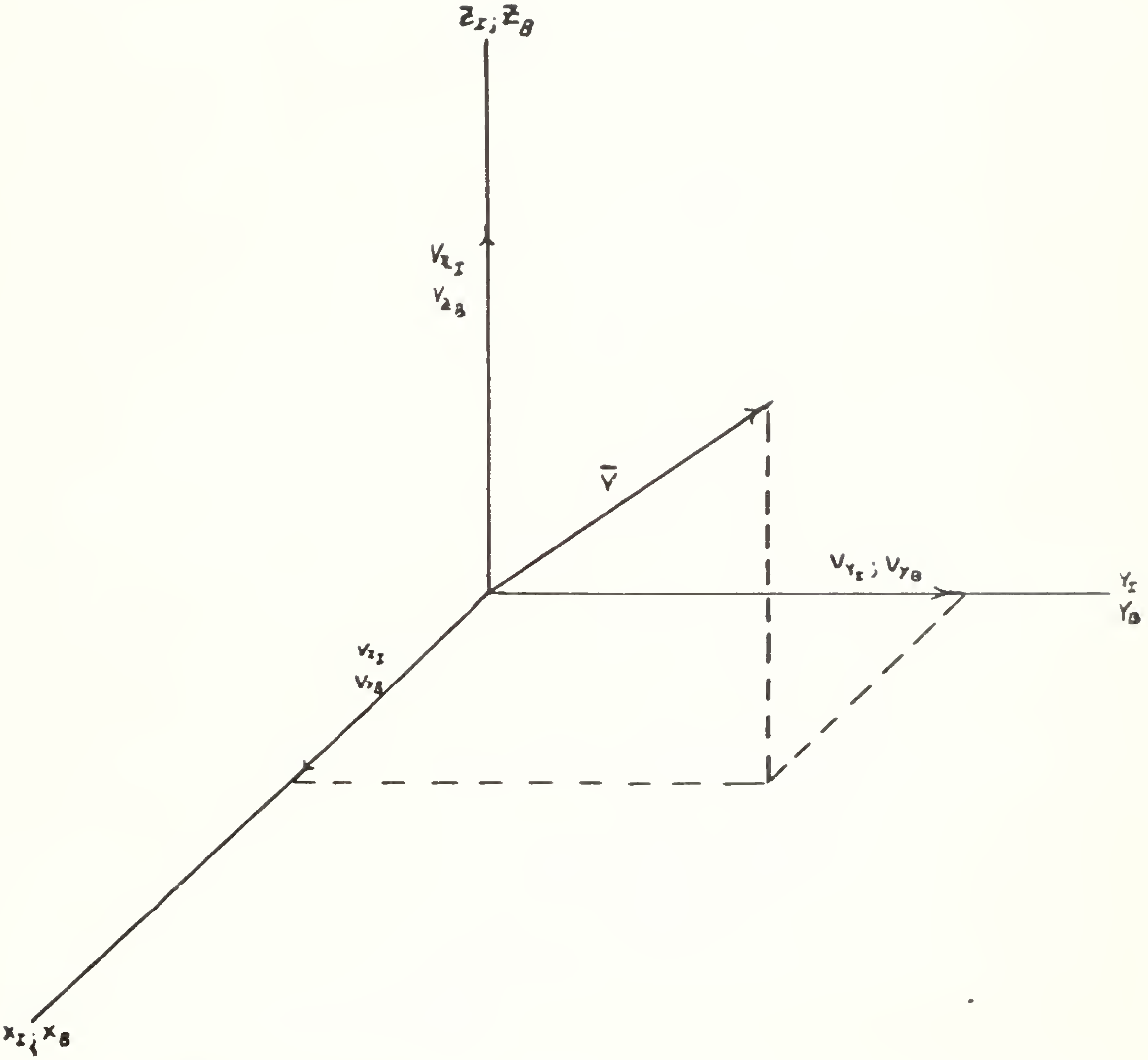
(A-9)

$$\begin{bmatrix} V_{x_I} \\ V_{y_I} \\ V_{z_I} \end{bmatrix} = \begin{bmatrix} 1 & -A_{z_I} & A_{y_I} \\ A_{z_I} & 1 & -A_{x_I} \\ -A_{y_I} & A_{x_I} & 1 \end{bmatrix} \begin{bmatrix} V_{x_B} \\ V_{y_B} \\ V_{z_B} \end{bmatrix}$$

It will be noted that the restriction imposed on the size of the angles in (A-9) is more severe than that imposed by the usual small angle assumption. Therefore replacing the sine of the angle by the angle and the cosine by unity is justified.

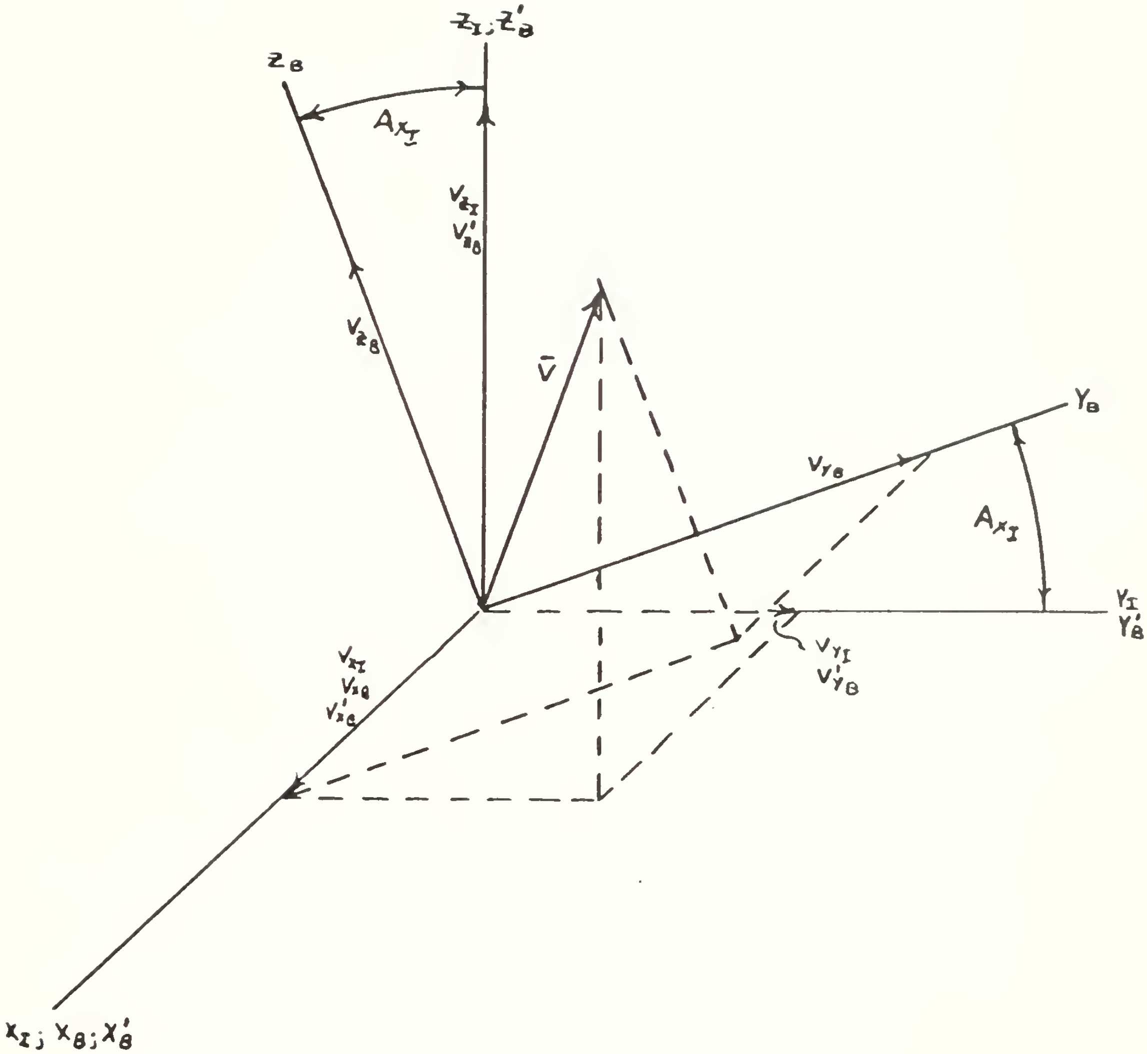
It was noted above that the transformation given by (A-8) is not unique. In fact there are six different matrices similar to the one in (A-8) corresponding to the six orders of rotation possible about the inertial axes. However, under the small angle assumption made

these all reduce to (A-9). The equation (A-9) and (2-2) are identical.



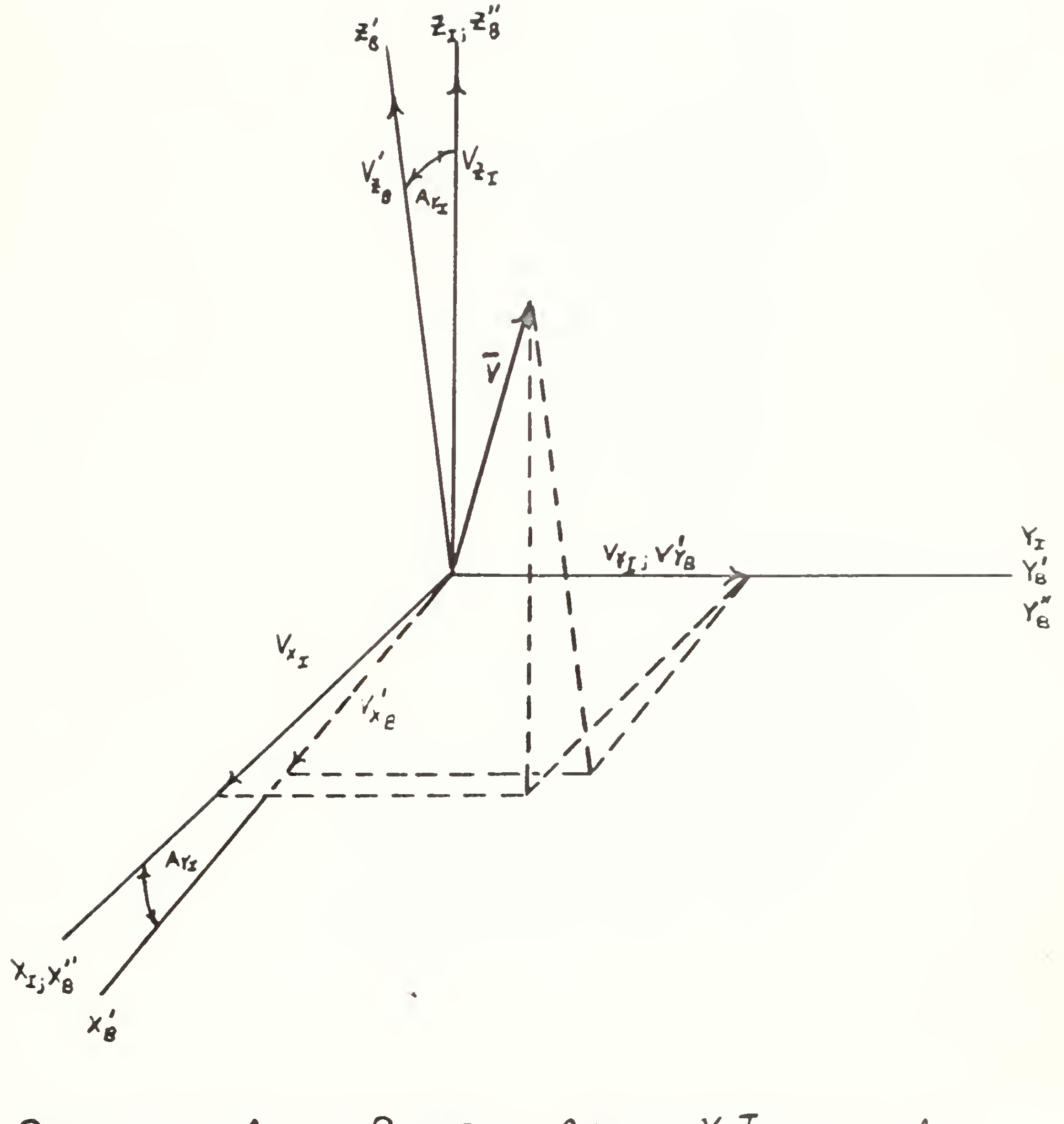
INITIAL ORIENTATION OF BODY AXES, INERTIAL AXES, AND VECTOR $\bar{V}$

Fig A-1



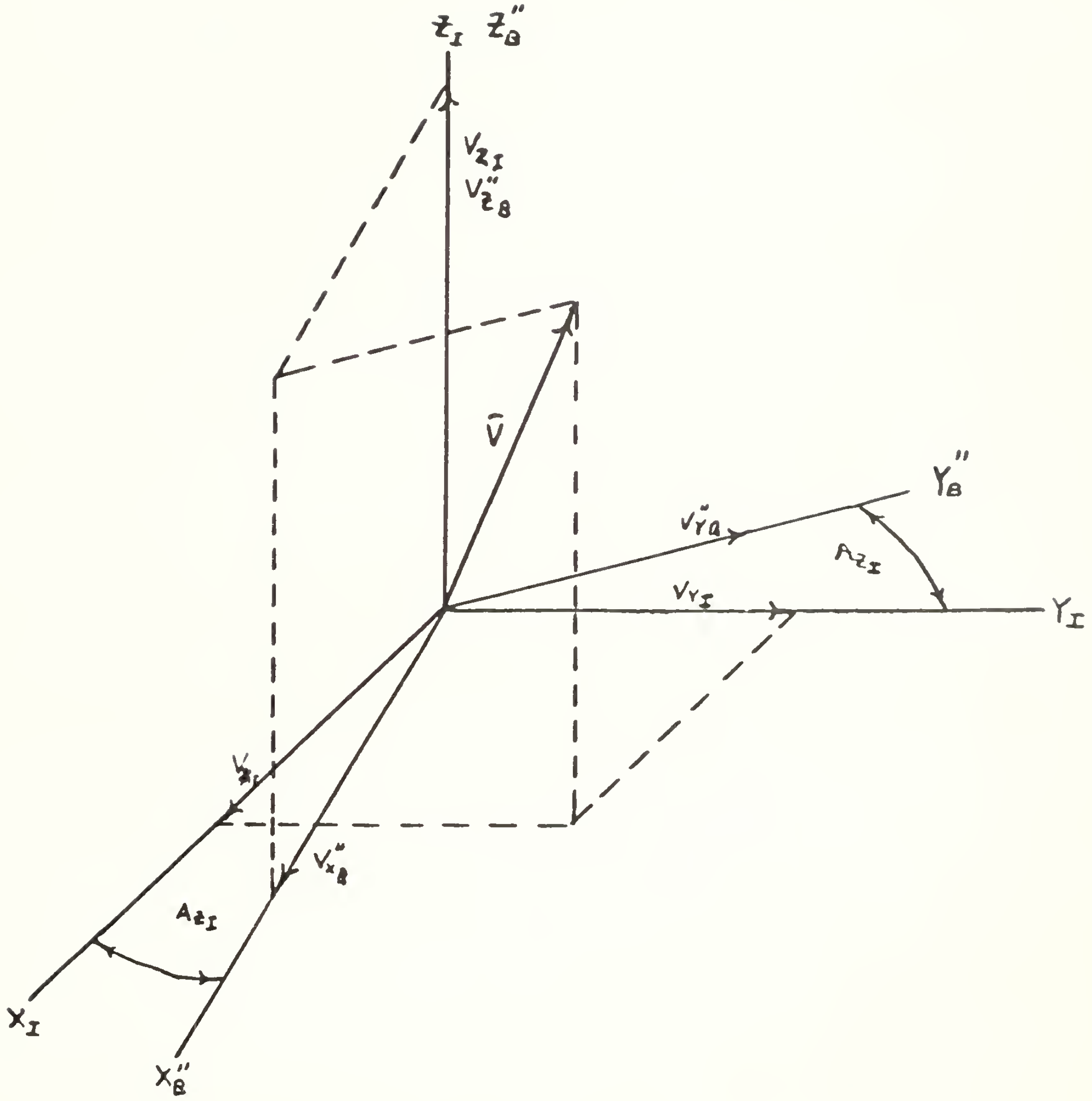
ORIENTATION AFTER ROTATION ABOUT X INERTIAL AXIS

Fig. A-2



ORIENTATION AFTER ROTATION ABOUT Y INERTIAL AXIS

Fig A-3



FINAL ORIENTATION AFTER ROTATION ABOUT Z INERTIL AXES

Fig A-4

APPENDIX B

SYSTEM PERFORMANCE EQUATIONS

The performance equations for the system of Fig. 2-1 are developed below in the order of their discussion in Chapter III.

The missile is taken as a rigid body. Since translational motion is of no interest in this analysis, the center of mass of the missile is assumed fixed in inertial space. The missile equations of motion are then obtained by summing moments about the missile center of gravity. Expressing components in body axes gives Euler's equations:

(B-1)

$$\dot{W}_x + (I_z - I_y)\, W_z W_y = M_x$$

$$\dot{W}_y + (I_x - I_z)\, W_x W_z = M_y$$

$$\dot{W}_z + (I_y - I_x)\, W_y W_x = M_z$$

where in the present case the sum of the applied moments on the right above includes the restoring torque applied to the missile by the control system and the interfering torques due to missile angular vibration. Thus:

(B-2)

$$M_x = M_{m_x} + M(int)_x$$

$$M_y = M_{m_y} + M(int)_y$$

$$M_z = M_{m_z} + M(int)_z$$

The remainder of the system performance equations are concerned with the determination of the restoring torque ($\overline{M}_m$) as a function of missile motion.

In the missile configuration being considered restoring torques for the pitch and yaw axes (y and z axes) are produced by rotating the thrust line of the missile engine away from the missile center of gravity. Restoring torque is roll is produced by two nozzles on the circumference of the missile at the opposite ends of a diameter. The components of restoring torque about the pitch and yaw axes may be written:

(B-3)

$$M_{m_y} = T\ l\ A_{m_y}$$

$$M_{m_z} = T\ l\ A_{m_z}$$

Where: T = the thrust of the missile motor, assumed constant.

l = distance from the missile center of gravity to the motor pivot point.

A_m = the angle through which the motor is rotated about the pitch or yaw axis (Assumed small)

Similarly the restoring torque in roll becomes:

(B-4)

$$M_{m_x} = T_{(re)}\ d$$

Where: $T_{(re)}$ = the thrust of the roll engine.

d = the diameter of the missile.

The system which positions the missile motor in pitch and yaw is assumed to be a second order system which receives gyro signals as input and positions the missile motor about the appropriate body axis.

The performance equations may be written:

(B-5)

$$\left[\left(\frac{p}{W_{n_{pcs}}}\right)^2 + \frac{2(DR)_{pcs}}{W_{n_{pcs}}}\,p + 1\right] A_{m_y} = S_{pcs(e_{igu};A)}e_{(igu)_y} + S_{pcs(e_{rgu};A)}e_{(rgu)_y}$$

$$\left[\left(\frac{p}{W_{n_{ycs}}}\right)^2 + \frac{2(DR)_{ycs}}{W_{n_{ycs}}} + 1\right] A_{m_z} = S_{ycs(e_{igu};A)}e_{(igu)_z} + S_{ycs(e_{rgu};A)}e_{(rgu)_z}$$

Where:

$(DR)_{pcs}$ = Pitch control system damping ratio.

$W_{n_{pcs}}$ = Pitch control system natural frequency.

$S_{pcs(e_{igu};A)}$ = Pitch control system sensitivity from integrating gyro signal to motor angle.

$S_{pcs(e_{rgu};A)}$ = Pitch control system sensitivity from rate gyro signal to motor angle.

Similar definitions hold for the yaw control system.

Roll engine thrust is assumed to be directly proportional to gyro signals. That is:

(B-6)

$$T_{(re)} = S_{rcs(e_{igu};T)}e_{igu_x} + S_{rcs(e_{rgu};T)}e_{rgu_x}$$

Equations (B-3) and B-5 may be combined to give the restoring torques in pitch and yaw as a function of gyro signal. Equations(B-4) and (B-6) may be similarly combined. Then:

(B-7)

$$M_{m_x} = S_{(rcs)(e_{igu};M)}e_{igu_x} + S_{(rcs)(e_{rgu};M)}e_{rgu_x}$$

$$\left[\left(\frac{p}{W_{n_{pcs}}}\right)^2 + \frac{2(DR)_{pcs}}{W_{n_{pcs}}} p + 1\right] M_{m_y} = S_{pcs(e_{igu};M)} e_{igu_y} + S_{pcs(e_{rgu};M)} e_{rgu_y}$$

$$\left[\left(\frac{p}{W_{n_{ycs}}}\right)^2 + \frac{2(DR)_{ycs}}{W_{n_{ycs}}} p + 1\right] M_{m_z} = S_{ycs(e_{igu};M)} e_{igu_z} + S_{ycs(e_{rgu};M)} e_{rgu_z}$$

Where: $S_{(rcs)(e_{igu};M)} = d\, S_{(rcs)(e_{igu};T)}$

$S_{(ycs)(e_{igu};M)} = T\, 1\, S_{(ycs)(e_{igu};A)}$

The remaining sensitivities have been similarly redefined.

In writing equations to describe the gyro units, the gyro uncertainties are neglected due to the relatively short system operating time. The rate gyro units are assumed to be proportional mechanisms so that:

(B-8)

$$e_{rgu_x} = S_{rgu(W;e)}\, W_x$$

$$e_{rgu_y} = S_{rgu(W;e)}\, W_y$$

$$e_{rgu_z} = S_{rgu(W;e)}\, W_z$$

where the sensitivity is the rate gyro sensitivity from input axis angular velocity to output signal.

The performance equations for the integrating gyro is developed in Derivation Summary 2 of Reference 2. In terms of gyro input, output and spin reference axes, the torque summation on the gyro gimbal is:

(B-9)

$$-I_{(gim)}\ \ddot{A}_{(gim)} + \ddot{A}_{(I-B)CA} - A_{(vd)(\dot{A};M)}\ \dot{A}_{(gim)}$$

$$+H\ \dot{A}_{(I-B)(IA)}\cos A_{(gim)} - H\ \dot{A}_{(I-B)(SRA)}\sin A_{(gim)} = 0$$

The orientation of the gyros with respect to body axes is given in the following Table:

Gyro	Input Axis	Spin Reference Axis	Output Axis
Roll	X	Y	Z
Pitch	Y	X	-Z
Yaw	Z	Y	-X

Table B-1

Gyro Orientations

where the entries in the table refer to the missile body axes.

The relationship between gyro gimbal angle and gyro output signal is:

(B-10)

$$e_{igu} = S_{(igu)(A_{gim};e)}\ A_{gim}$$

(B-9) and (B-10) may be combined to give the equation for gyro output signal. Equations may be written for each axis by replacing the subscripts refering to gyro axes by the appropriate body axis subscripts from the Table (B-1). Then, after rearranging (B-9) becomes:

(B-11)

$$I_{(gim)}\ddot{e}_{(igu)_x} + S_{(vd)(W;M)}\dot{e}_{(igu)_x} + H\ W_y\ e_{(igu)_x}$$

$$= H\ S_{(Sg)(A;e)}\ W_y - I_{(gim)}S_{(sg)(A;e)}\ \dot{W}_z$$

$$I_{(gim)} \ddot{e}_{(igu)_y} + S_{(vd)(W.M)} \dot{e}_{(igu)_y} + H W_x e_{(igu)_y}$$

$$= H S_{(Sg)(A;e)} W_y + I_{(gim)} S_{(Sg)(A;e)} \dot{W}_z$$

$$I_{(gim)} \ddot{e}_{(igu)_z} + S_{(vd)(W;M)} \dot{e}_{(igu)_z} + H W_y e_{(igu)_z}$$

$$= H S_{(Sg)(A;e)} W_z + I_{(gim)} S_{(Sg)(A;e)} \dot{W}_x$$

The equations (B-1), (B-2), (B-7) and (B-11) completely describe the performance of the proposed system. These can be simplified by redefining certain of the constants.

Define:

(B-12)

$$(IR)_x = \frac{I_y - I_z}{I_x}$$

$$(IR)_y = \frac{I_z - I_x}{I_y}$$

$$(IR)_z = \frac{I_x - I_y}{I_z}$$

Also let:

(B-13)

$$\frac{M_{(int)_x}}{I_x} = \ddot{A}_{(int)_x} \; ; \quad \frac{M_{m_x}}{I_x} = \ddot{A}_{m_x}$$

$$\frac{M_{(int)_y}}{I_y} = \ddot{A}_{(int)_y} \; ; \quad \frac{M_{m_y}}{I_y} = \ddot{A}_{m_y}$$

$$\frac{M_{(int)_z}}{I_z} = A_{(int)_z} \; ; \quad \frac{M_{m_z}}{I_z} = A_{(int)_z}$$

The gyro angular momentum and the viscous damper sensitivity are equal for the gyro being used so that:

(B-14)

$$(CT)_{igu} = \frac{I_{gim}}{S_{(vd)(W;M)}} = \frac{I_{gim}}{H}$$

Substitution of (B-12) (B-13) (B-14) into the system equations has the effect of lumping the system sensitivity for each axis in the torque generating system equations, except for the signal generator sensitivity. If the gyro equations are expressed in terms of gyro gimbal angle instead of voltage this sensitivity is also carried up into the torque generating system. The performance equations for the systems then become:

(B-15)

$$\dot{W}_x - (IR)_x W_y W_z = \ddot{A}_{(int)_x} + \ddot{A}_{m_x}$$

$$\dot{W}_y - (IR)_y W_x W_z = \ddot{A}_{(int)_y} + \ddot{A}_{m_y}$$

$$\dot{W}_z - (IR)_z W_x W_y = \ddot{A}_{(int)_z} + \ddot{A}_{m_z}$$

$$\ddot{A}_{m_z} = S_{(rcs)(A_{gim};\ddot{A})} A_{gim_x} + S_{rcs(W;\ddot{A})} W_x$$

$$\left[\frac{p}{W_{n_{pcs}}}^2 + \frac{2(DR)_{pcs}}{W_{n_{pcs}}} p + 1\right] \ddot{A}_{m_y} = S_{pcs(A_{gim};\ddot{A})} A_{gim_y} + S_{pcs(W;\ddot{A})} W_y$$

$$\left[\frac{p}{W_{n_{ycs}}}^2 + \frac{2(DR)_{ycs}}{W_{n_{ycs}}} + 1\right] \ddot{A}_{m_z} = S_{ycs(A_{gim};\ddot{A})} A_{gim_z} + S_{pcs(W;\ddot{A})} W_z$$

$$(CT)_{igu} \ddot{A}_{gim_x} + \dot{A}_{gim_x} + W_y A_{gim_x} = W_x - (CT)_{igu} \dot{W}_z$$

$$(CT)_{igu} \ddot{A}_{gim_y} + A_{gim_y} + W_x A_{gim_y} = W_y + (CT)_{igu} \dot{W}_z$$

$$(CT)_{igu} A_{gim_z} + A_{gim_z} + W_y A_{gim_z} = W_z + (CT)_{igu} \dot{W}_x$$

Where: $$S_{(rcs)(A_{gim};A)} = \frac{S_{(rcs)(e_{igu};M)} \; S_{(Sg)(A;e)}}{I_x}$$

$$S_{(rcs)(W;A)} = \frac{S_{(rcs)(e_{rgu};M)}\ S_{(rgu)(W;e)}}{I_x}$$

and identical definitions for the sensitivities hold for the remaining axes of the system.

The equations (B-15) are the set that was solved on the digital computer in the present analysis. The values chosen for the various constants in these equations are given in Table 3-1.

Reduced Dynamics Case

The simplified system which was discussed in Chapter III can be obtained from the equations (B-15) by eliminating the dynamics and the non-linearities from the last six equations. These equations can then be combined with the missile equations of motion to give:

(B-16)

$$\dot{W}_x - (IR)_x W_y W_z = \ddot{A}_{(int)_x} + S_{(rcs)(A_{gim};\ddot{A})}\ A_{(gim)_x} + S_{(rcs)(W;\ddot{A})}\ W_x$$

$$\dot{W}_y - (IR)_y W_x W_z = \ddot{A}_{(int)_y} + S_{(pcs)(A_{gim};\ddot{A})}\ A_{(gim)_y} + S_{(pcs)(W;\ddot{A})}\ W_y$$

$$\dot{W}_z - (IR)_z W_x W_y = \ddot{A}_{(int)_z} + S_{(ycs)(A_{gim};\ddot{A})}\ A_{(gim)_z} + S_{(pcs)(W;\ddot{A})}\ W_z$$

Where:

$$\dot{A}_{(gim)_x} = W_x$$

$$\dot{A}_{(gim)_y} = W_y$$

$$\dot{A}_{(gim)_z} = W_z$$

The set of equations (B-16) was the set solved for the system with simplified dynamics. The constants here identical with those for the complete system which are given in Table 3-1.

Single Axis, Linearized Stability Analysis

The question of system stability was approached by making a root locus plot of the linearized performance equations for the yaw axis. The yaw axis equations may be written:

(B-17)

$$\dot{W}_z - .925\, W_x W_y + \ddot{A}_{(int)_z} + \ddot{A}_{m_z}$$

$$\left[\frac{p^2}{625} + \frac{p}{25} + 1\right] \ddot{A}_{m_z} = S_{ycs(A_{gim}\ddot{A})} \left[A_{(gim)_z} - \frac{S_{ycs(W:\ddot{A})}}{S_{ycs(A_{gim}\ddot{A})}} W_z \right]$$

$$\left[\frac{p^2}{333} - p - W_y\right] A_{gim_z} = W_z + \frac{\dot{W}_x}{333}$$

where numbers have been substituted from Table 3-1 for all except the sensitivities. These may be combined to give a single expression provided that the character of the non-linearities is preserved. That is, the order of factors in the non-linear expression must be maintained. Then by substituting from the first and third equation into the second and their results:

(B-18)

$$\left[\frac{p^2}{333} + p + W_y\right]\left[\frac{p^2}{625} + \frac{p}{25} + 1\right]\left[\dot{W}_z - .925 W_x W_y - \ddot{A}_{(int)_x}\right]$$

$$= S_{(pcs)(A_{gim};\ddot{A})} \left\{ \frac{\dot{W}_x}{333} + \left[1 + .1985\left(\frac{p^2}{333} + p + W_y\right)\right] W_x \right\}$$

where the ratio:

(B-19)

$$\frac{S_{ycs(W;A)}}{S_{ycs(A_{gim};A)}} = .1985$$

is arbitrarily chosen

If it is now assumed that there is no motion in roll and pitch then:

(B-20)

$$W_x = W_y = 0$$

The equation (B-18) then becomes linear and the performance function for the yaw axis may be written:

(B-21)

$$[PF]_{ycs(A_{int};W_x)}$$

$$= \frac{\left[\frac{p^2}{333} + p\right]\left[\frac{p^2}{625} + \frac{p}{25} + 1\right]}{p^2\left[\frac{p}{333} + 1\right]\left[\frac{p^2}{625} + \frac{p}{25} + 1\right] + S_{ycs(A_{gim};A)}\left[\frac{.1985}{333}p^2 + .1985p + 1\right]}$$

and the characteristic equation is:

(B-22)

$$1 - \frac{S_{ycs}\left(.1985 \times .003\, p^2 + .1985\, p + 1\right)}{p^2\left[.003p + 1\right]\left[\frac{p^2}{625} + \frac{p}{25} + 1\right]} = 0$$

$$= 1 - (PF)_{open\ loop} = 0$$

The complex frequency plot of the open loop performance function $(PF)_{ol}$ as a function of $S_{ycs(A_{gim};A)}$ is given in Fig. 3-3. The plots and zeros of this plot are given below.

Poles	Source
$P_1 = -333.3$	Integrating Gyro
$P_2, P_3 = 0$	Integrating Gyro and Missile Equation
$P_4, P_5 = -12.5 \pm j\,21.65$	Yaw Torque Generating System
Zeros	
$Z_1 = -328.9$	Rate Feedback
$Z_2 = 5.1$	Rate Feedback

As the open loop sensitivity is increased from zero it will be observed that the pole pair at the origin move to the left while the pole pair due to the torque generating system move to the right. The choice of the open loop sensitivity $S_{ycs(A_{gim};A)}$ was made so as to give both of these oscillatory modes the same damping ratio. This means that both poles must lie on the same radial line from the origin. By trial and error this radial was determined to be the line for a damping ratio of 0.44. When both pole pairs lie on this line the open loop sensitivity was determined as:

(B-23)

$$S_{ycs(A_{gim};A)} = 28.0$$

which is the value shown in Table 3-1.

APPENDIX C

COMPUTER PROGRAM

The following program is the complete program used for the system, with random forcing function. The program is coded using the [illegible] system(14)(15). Other programs used are simply modifications of this basic program.

```
HEADING          15-13-58        000000030    COMP PROGRAM
COMPLETE PROGRAM FOR RANDOM FORCING FUNCTION.

INITIAL CONDITION READ-IN. PROGRAM RESET SUBROUTINES

AA005          TRNO   0            0
0A010          BLOCK  1            79
0A015  B1      TRNO   [illegible]
0A020          BIT    0     +20    S1
0A025  B1      TS     1
0A030  S1      SETEO  22
0A035          BIT    110  +16
0A040          R      2
0A045          RAR    2                    INITIAL CONDITION READ IN
0A050          RAB    [illegible]          READ CARD TO S1
0A055  R1     -RD     52                   TRANS TO D1 VIA THE BUFFER
0A060          SET    ,54                  LOADS T DELTA T AND Y ZEROS
0A065          LD     52                   AT SIX WORDS PER CARD
0A070          SET    ,15
0A075          STB    D1         B          D1 IS FILE. BAND
0A080          AB     6
0A081          BMNA   N4
0A082          SA     1            R1
0A083  N4      RD     52
0A084          SET    ,057
0A085          LB     52
0A086          SET    2057
0A087          STD    ,2
0A088          RD     52
0A089          SET    ,057
0A090          LB     52
```

```
00000 HEADING         09-13-58        0000 0000   COMP PROGRAM
0A091       SET   0057
0A092       STR   C4
0A095 L1    +00000 [illegible]                                  + D


      RIGHT HAND SIDE COMPUTATION. [illegible] EQUATIONS         +


0A100 N3    SET   9019               START OF BUFFER LOAD
0A105       LB    800                 H IS 9019 FOR D1
0A110       LB    938                 H IS 9000 FOR A1
0A115       LB    C2
0A120 R7    RAU   D1    +4H          COMPUTE W.X AND AUX         WZ
0A125       STU   D1    +27H           WZ TO [illegible]
0A130       FM    D1    +3H          WZ X WY
0A135       FM    A1       H           (IR) X WZ X WY
0A140       FA    C2       H           + FORCING FUNCTION
0A145       FA    D1    +10H         + MOTOR ACC
0A150       STU   D1    +22H           W.X   (F)
0A155       RAU   D1    +2H          COMPUTATION OF W.Y          WX
0A160       STU   D1    +25H
0A165       FM    D1    +4H            WX X WZ
0A170       FM    A1    +1H            (IR) WX WZ
0A175       FA    C2    +1H            FORCING FUNCTION
0A180       FA    D1    +11H            +MOTOR ACC
0A185       STU   D1    +23H            W.Y
0A190       RAU   D1    +3H            WY
0A195       STU   D1    +26H
0A200       FM    D1    +2H            WY WY
0A205       FM    A1    +2H                                      (IR)
0A210       FA    C2    +2H          +FORCING FN.
```

```
00000  HEADING          05-13-58        0000 0000  COMP PROGRAM
0A215       STU   D1   +24H
0A220       RAU   D1   +17H           A-GIM
0A225       FM    A1    +3H                              CONST COEFF
0A230       FA    D1   +24H
0A235       STU   D1   +24H
0A240       RAU   D1    +4H             AZ
0A245       FM    A1    +4H             CONST COEFF
0A250       FA    D1   +24H
0A255       STU   D1   +24H  58          A.Z                            ++


       COMPUTATION OF RIGHT HAND SIDES FOR TORQUE GENERATING SYSTEM     +


0A260 R8    RAU   D1    +8H           COMMENCE AM FOURTH COMPUTATION  AMX3
0A265       STU   D1   +30H
0A270       FM    A1    +5H           COEFF
0A275       STU   D1   +28H
0A280       RAU   D1   +10H           AMX2
0A285       FM    A1    +6H           COEFF
0A290       FA    D1   +28H
0A295       STU   D1   +28H
0A300       RAU   D1   +15H          A-GIM X
0A305       FM    A1    +7H             COEFF
0A310       FA    D1   +28H
0A315       STU   D1   +28H
0A320       RAU   D1    +2H             AX
0A325       FM    A1    +8H           COEFF
0A330       FA    D1   +25H
0A335       STU   D1   +28H           FINAL STORAGE
0A340       RAU   D1    +9H             COMMENCE AMY FOURTH
```

```
00000  HEADING           05-13-58     [illegible]  COMP PROGRAM
0A345         STU   D1  +31H
0A350         FM    A1  +5 H          COEFF
0A355         STU   D1  +27H
0A360         RAU   D1  +11H          AWYZ
0A365         FM    A1  +6 H          COEFF
0A370         FA    D1  +29H
0A375         STU   D1  +29H
0A380         RAU   D1  +16H          A-GIM Y
0A385         FM    A1  +4 H          COEFF
0A390         FA    D1  +29H
0A395         STU   D1  +29H
0A400         RAU   D1  +3 H          WY
0A405         FM    A1  +10H          COEFF
0A410         FA    D1  +29H
0A415         STU   D1  +29H  R7      AMY--- STORED (F)
```

COMPUTATION OF RIGHT HAND SIDES FOR GYRO EQUATIONS

```
0A420  R9     RAU   D1  +12H          START GYRO COMPT
0A425         STU   D1  +35H
0A430         RAU   D1  +15H          A-GIM X
0A435         FM    D1  +3 H          W Y
0A440         FA    D1  +2 H          WX
0A445         FS    D1  +35H
0A450         FM    A1  +11H          COEFF
0A455         FA    D1  +24H          W.Z
0A460         STU   D1  +32H          X GYRO COMPLETE
0A465         RAU   D1  +13H          COMMENCE Y GYRO
0A470         STU   D1  +36H
```

```
00000  HEADING             05-13-58        1000 DOS   COMP PROGRAM
0A475        RSU   D1   +16H               A-GIMY
0A480        FM    D1   +4 H               A Z
0A485        FA    D1   +3 H               A Y
0A490        F     D1   +36H               A.GIM Y
0A495        FM    A1   +11H               COEFF
0A500        FA    D1   +22H               A.X
0A505        STU   D1   +33H               Y GYRO COMP
0A510        RAU   D1   +14H               A.GIMZ
0A515        STU   D1   +37H
0A520        RSU   D1   +17H               A-GIMZ
0A525        FM    D1   +  H               Y
0A530        FA    D1   +4 H               AZ
0A535        FD    D1   +37H               A.GIMZ
0A540        FM    A1   +11H               COEFF
0A545        FS    D1   +22H               A.X
0A550        STU   D1   +34H   R10         Z GYRO COMP
```

COMPUTATION OF RIGHT HAND ... FOR TRANSFORMATION EQUATIONS

```
0A555 R10    RAU   D1   +4 H               COMMENCE INERTIAL TRANSFORM AZ
0A560        FM    D1   +1 H               AXI
0A565        STU   D1   +38H
0A570        RSU   D1   +3 H               AY
0A575        FM    D1   +20H               AZI
0A580        FA    D1   +38H
0A585        FA    D1   +2 H               AX
0A590        STU   D1   +38H               A.XI COMP
0A595        RSU   D1   +4 H               AZ
0A600        FM    D1   +18H               AXI
```

```
00000  HEADING        05-13-58      0000 0000   COMP PROGRAM
0A605      STU   D1   +39H
0A610      RAU   D1   +2 H          AX
0A615      FM    D1   +20H          AZI
0A620      FA    D1   +3 H
0A625      FA    D1   +5 H          AY
0A630      STU   D1   +39H          A.YI COMP
0A635      RSU   D1   +2 H          AX
0A640      FM    D1   +19H          AYI
0A645      STU   D1   +40H
0A650      RAU   D1   +3 H          AY
0A655      FM    D1   +18H          AXI
0A660      FA    D1   +4 H          AZ
0A665      FA    D1   +40H
0A670      STU   D1   +40H          A.ZI COMP
```

PUNCH ROUTINE FOR RECOVERING INITIAL CONDITIONS

```
0A675      SET   9019
0A680      STB   800
0A685      STB   938
0A690      STB   C2
0A695      RAU   R1
0A700      BRMIN 8000        E2     SET 8000 TO 00NNNN0123
0A705 123  RAL   A2                 BRING IN PUNCH COUNTER
0A710      SL    L1  +1
0A715      STL   A2
0A720      RAABL 8000
0A725      SRT   4
0A730      AL    A2                 COMPARE  COUNTER WITH DA OF 8000
```

```
00000  HEADING          05-13-58        0000 0000   COMP PROGRAM
0A735       BRMIN R15          R13                                              +


0A740 R15   STU   A2
0A745       RAU   8000
0A750       BRMIN R16
0A755       SET   9057
0A760       LB    C2                       THIS IS THE LONG PUNCH
0A765       SET   9057
0A770       STB   28
0A774       PCH   27
0A775       SET   9057                     PUNCH FF TIME COUNTERS
0A776       LB    C4
0A777       SET   9057
0A778       STB   28
0A779       PCH   27
0A780       RAA   0800
0A785       RSB   7
0A790 R17   SET   9054
0A795       LB    0000    A
0A800       SET   9054
0A805       STB   0078
0A810       RAU   8005
0A815       STU   0077
0A820       PCH   0077
0A825       AB    1
0A830       AA    6
0A835       BNZB  R17          R13       END OF LONG PUNCH
0A840 A2    +0000000000                                                         D
0A845 34    +0000999000+0000000000+0080800000                                   D
0A850 27    +0000000101                                                         D
0A855 84    +0000999990+0000000000+0080800000                                 + D
```

```
00000  HEADING          05-13-58      0000 0000    COMP PROGRAM
                           PUNCH ROUTINE FOR DATA

0A860 R16   RAU    800                   DELTA T TO CARD NO
0A861       STU    128
0A862       FASN   F3
0A863 F3    +0000000054
0A864       SRT    4
0A865       STU    177
0A866       STU    227
0A867       STU    277
0A870       LD     801
0A875       STD    129
0A880       PCH    127
0A885       SET    9057
0A890       LB     C2                  A DOUBLE DOT INTERFERING
0A895       SET    9057
0A900       STB    178
0A901       RAU    135                    0135 IS 2-92
0A902       AU     L1    +1
0A903       STU    181
0A905       PCH    177                 THIS IS THE SHORT PUNCH
0A910       RSA    3
0A915 P1    RAU    01    +18  A
0A920       FASN   F1
0A925       STU    181        A
0A930       AA     1
0A935       BNZA   P1
0A936       RAU    181
0A937       AU     L1    +1
0A938       STU    181
0A940       PCH    177
0A945       RSA    3
```

```
00000  HEADING            05-15-58        0000 0000   COMP PROGRAM
0A950 P2   RAU   D1   +8   A
0A955      FASN  F1
0A960      STU   181       A
0A965      AA    1
0A970      BNZA  P2
0A971      RAU   181
0A972      AU    L1   +1
0A973      STU   181
0A975      PCH   177
0A980      RSA   3
0A985 P3   RAU   D1   +21  A
0A990      FASN  F1
0A995      STU   231       A
1A005      AA    1
1A010      BNZA  P3
1A011      RAU   181
1A012      AU    L1   +1
1A013      STU   231
1A015      PCH   227
1A020      RSA   3
1A025 P4   RAU   D1   +21  A
1A030      FS    D1   +8   A
1A035      FASN  F1
1A040      STU   181       A
1A045      AA    1
1A050      BNZA  P4
1A051      RAU   231
1A052      AU    L1   +1
1A053      STU   181
1A055      PCH   177
1A060      RSA   3
1A065 P5   RAU   D1   +8   A
1A070      FS    D1   +18  A
1A075      FASN  F1
```

```
00000  HEADING          05-13-58         0000 0000   COMP PROGRAM
1A080       STU    181       A
1A085       AA     1
1A090       BNZA   P5
1A091       RAU    181
1A092       AU     L1   +1
1A093       STU    181
1A095       PCH    177
1A100       RSA    3
1A105 P6    RAU    D1   +21  A
1A110       FS     D1   +18  A
1A115       FASN   +1
1A120       STU    281       A
1A121       RAU    181
1A122       AU     L1   +1
1A123       STU    281
1A125       AA     1
1A130       BNZA   P5
1A135       RAU    A3   +2
1A140       SU     L1   +1
1A145       STU    A3   +2
1A150       AU     A3
1A155       BRNZU  P7
1A160       RAU    286
1A165       AU     A3   +1
1A170       STU    286
1A175       PCH    277
1A180       RAU    286
1A181       STL    A3   +2
1A185       SU     A3   +1
1A190       STU    286          R13
1A195 P7    PCH    277          R13
1A200 A3    +0000000003+0000000028+0000000000                        D
1A205 F1    +0000000050                                            + D
```

```
00000  HEADING          05-13-58        0000 0000    COMP PROGRAM
             COPY OF PUNCH ROUTINE FOR TAPE STORAGE OF DATA

1A210 0124 RAL    A2
1A215      SL     L1   +1
1A220      STL    A2
1A225      RAABL  8000
1A230      SRT    4
1A235      AL     A2
1A240      BRMIN  R18           R13
1A241 R18  RAU    800
1A242      STL    A2
1A246      STU    128
1A247      FASN   F4
1A249      SRT    4
1A250      STU    177
1A251      STU    227
1A252      STU    277
1A255      LD     801
1A260      STD    129
1A265      RAL    Q1            222
1A270 Q1   DD     127           T1
1A275 T1   SET    9057
1A280      LB     C2                          A [illegible] NOT INTERFERING
1A285      SET    9057
1A290      STB    174
1A291      RAU    135                              0145 I[illegible] ZERO
1A292      AU     L1   +1
1A293      STU    181
1A295      RAL    Q2            222
1A300 Q2   DD     177           T2
1A305 T2   RSA    5
1A310 P8   RAU    [illegible]   +18 A
```

```
00000  HEADING          05-13-55        [illegible]  [illegible]  COMP PROGRAM
1A315        FASN   F1
1A320        STU    181        A
1A325        AA     1
1A330        BNZA   P8
1A331        RAU    181
1A332        AU     L1    +1
1A333        STU    181
1A335        RAL    Q3              222
1A340  Q3    00     177             T3
1A345  T3    RSA    3
1A350  P9    RAU    D1    +8   A
1A355        FASN   F1
1A360        STU    181        A
1A365        AA     1
1A370        BNZA   P9
1A371        RAU    181
1A372        AU     L1    +1
1A373        STU    181
1A375        RAL    Q4              222
1A380  Q4    00     177             T4
1A385  T4    RSA    3
1A390  P10   RAU    D1    +21  A
1A395        FASN   F1
1A400        STU    231        A
1A405        AA     1
1A410        BNZA   P10
1A411        RAU    181
1A412        AU     L1    +1
1A413        STU    231
1A415        RAL    Q5              222
1A420  Q5    00     227             T5
1A425  T5    RSA    3
1A430  P11   RAU    D1    +21  A
1A435        FS     D1    +8   A
```

```
00000  HEADING              05-14-58            [illegible]     COMP PROGRAM
1A440          FASN   F1
1A445          STU    181          A
1A450          AA     1
1A455          BNZA   P11
1A456          RAU    231
1A457          AU     L1    +1
1A458          STU    181
1A460          RAL    Q6                 222
1A465  Q6      00     177                T6
1A470  T6      RSA    3
1A475  P12     RAU    D1    +8    A
1A480          FS     D1    +18   A
1A485          FASN   F1
1A490          STU    181          A
1A495          AA     1
1A500          BNZA   P12
1A501          RAU    181
1A502          AU     L1    +1
1A503          STU    181
1A505          RAL    Q7                 222
1A510  Q7      00     177                T7
1A515  T7      RSA    3
1A520  P13     RAU    D1    +21   A
1A525          FS     D1    +18   A
1A530          FASN   F1
1A535          STU    281          A
1A536          RAU    181
1A537          AU     L1    +1
1A538          STU    281
1A540          AA     1
1A545          BNZA   P13
1A550          RAU    A3    +2
1A555          SU     L1    +1
1A560          STU    A3    +2
```

```
00000  HEADING          05-13-58        0000 0000   COMP PROGRAM
1A565        AU     A3
1A570        BRNZU  P14
1A575        RAU    286
1A580        AU     A3   +1
1A585        STU    286
1A590        RAL    Q8          Z22
1A595  Q8    00     277         T8
1A600  T8    RAU    286
1A601        STL    A3   +2
1A605        SU     A3   +1
1A610        STU    286         R13
1A615  P14   RAL    Q9          Z22
1A620  Q9    00     277         R13
```

TAPE STORAGE SUBROUTINE

```
1A635  Z22   LD     3R                         STORE ANSWER ON TAPE 2 Z22-A
1A640        STDA   3R
1A645        SET    9950        8001
1A650  3R    LBB    0
1A655        AU     8R
1A660        AU     1R
1A665        STU    8R
1A670  11R   STD    9043
1A675        96     8012
1A680        25     8002
1A685        54     11R         12R
1A690  8R    +0000000000
1A695  1R    +0000000001
```

```
00000  HEADING         05-13-58        0000 0000    COMP PROGRAM
1A700 12R  STOP      2        11R                                    +


             PUNCH CONTROL WORDS AND BLOCK ORDERS                    +


1A705 134   +0000990000+0000000000+0080800008                         D
1A710 184   +0000999000+0000000000+0080800008                         D
1A715 234   +0000999000+0000000000+0080800008                         D
1A720 284   +0000999000+0000000000+0080800008                         D
1A725       ERASE 28          33
1A730       ERASE 51          60
1A735       ERASE 77          83
1A740       ERASE 127         133
1A745       ERASE 177         183
1A750       ERASE 227         233
1A755       ERASE 277         283                                    +


             ENTRY  TO DIFFERENTIAL EQUATION SUBROUTINE              +


1A760 R13   RSU    R1
1A765       STU    R1         E2
1A770 E2    RAL    E3
1A775       LD     R12        26        26 IS THE ENTRY TO DIFFEQ
1A780 E3    19     0800       N3                                     +
```

```
00000  HEADING        05-15-58        0005 0000    COMP PROGRAM
1A819 N1    +0000000049-0000000049+[illegible]                    D
1A820 C4    +0000000012+[illegible]                             + D


1A865 R11   RSU    R1
1A870       STU    R1          N3
1A875 76    IS     1898                                         +


1A880       BLOCK 0800         0890
1A885 H     IS     9012
1A890 D1    IS     0800                                         +


            NUMERICAL COEFFICIENT  TURALE                       +


1A895       BLOCK 0938         0949
1A900 H     IS     9000
1A905 A1    +9250000050-8912000050-1781000050-15[illegible]51-3110000051-2500000052  D
1A910 6R    -6250000053-1750000055-3460000054-1710000055-349[illegible]00054+3333333353+ D


1A915       BLOCK 0896         0899
1A920 H     IS     9012
1A925 C2    +4000000049+0000000000+[illegible]0+0000000020      D
1A930       FINIS S1                                            ++
```

```
00000  HEADING        05-13-58       0000 0000   COMP PROGRAM
                    FORCING FUNCTION GENERATING SECTION

1A784 R12   RSA    3                       BEGIN FORCING FUNCTION MODIFICATION
1A785 R21   RAU    C4    +3   A            C4 IS TIME COUNTER
1A786       SU     L1    +1                MODIFY TIME COUNTER
1A787       STU    C4    +3   A
1A788       BRMIN  R20            R22      IF C4 MINUS CHANGE FORCING FUNCTION
1A789 R20   RAABL  D1    +26  A            EXTRACT RANDOM NUMBER  FROM DIFEQ
1A790       SRT    7
1A791       RAU    8002
1A792       SLT    8
1A793       SRT    8
1A794       STU    C4    +3   A            STORE NEW TIME COUNTER
1A797       RAABL  D1    +17  A
1A798       SLT    2
1A799       RAU    8002
1A801       SU     N1    +2
1A802       BRMIN  R23                     BRANCH FOR + OR - ON NEW FF
1A803       RAABL  D1    +32  A            GENERATE POSITIVE FORCING FUNCTION
1A804       SRT    7
1A805       RAU    8002
1A806       SLT    9
1A807       AU     N1
1A808       STU    C2    +3   A  R22       STORE NEW FF AND GO OUT
1A809 R23   RSABL  D1    +32  A            GENERATE NEGATIVE FORCING FUNCTION
1A810       SRT    7
1A811       RAU    8002
1A812       SLT    9
1A813       AU     N1    +2
1A814       STU    C2    +3   A  R22       STORE NEGATIVE FF AND GO OUT
1A817 R22   AA     1                       GO TO NEXT AXIS OR EXIT
1A818       BNZA   R21            R11
```

APPENDIX D

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